

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Minimum Weight Spanning Trees



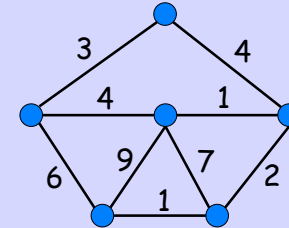
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mintree.1

6	9	13	7
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15	8	11	2

Minimum Spanning Trees

Suppose edges have weights:



Find **min weight** spanning tree?




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mintree.2

6	9	13	7
12	10	5	
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Build MST using **gray edges**

- Start with vertices, no edges
- Color components black & white
- Graph is connected, so have:
- gray edge ::= 
- add min weight gray edge

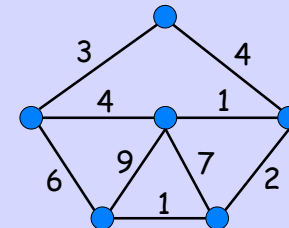


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mintree.3

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Minimum Spanning Trees color components



initial components:
isolated vertices

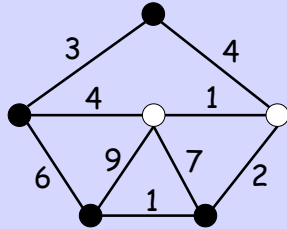


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mintree.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Minimum Spanning Trees color components



initial components:
isolated vertices

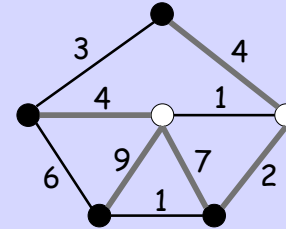


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mintree.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Minimum Spanning Trees gray edges

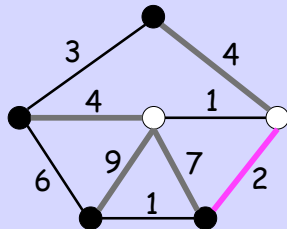


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mintree.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Minimum Spanning Trees gray edges: min weight



repeat:

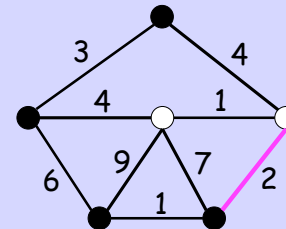


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mintree.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Minimum Spanning Trees re-color components

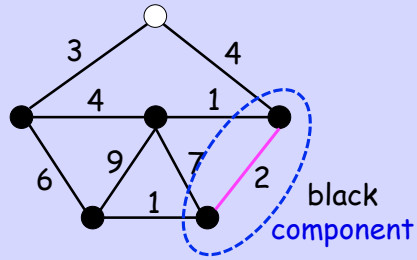


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mintree.8

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Minimum Spanning Trees re-color components

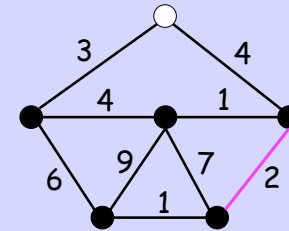


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mintree.9

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Minimum Spanning Trees re-color components



so gray edges must differ
from previously selected

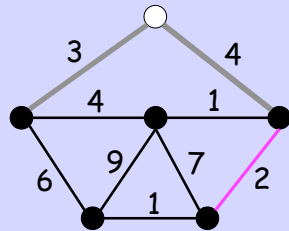


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mintree.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Minimum Spanning Trees gray edges

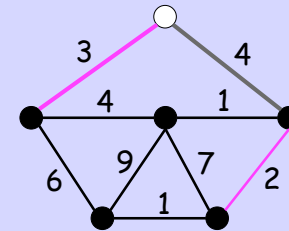


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mintree.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Minimum Spanning Trees gray edges: min weight



repeat:

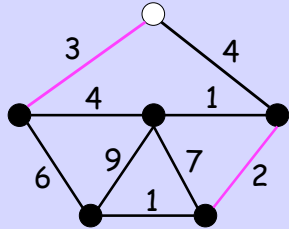


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mintree.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Minimum Spanning Trees re-color components

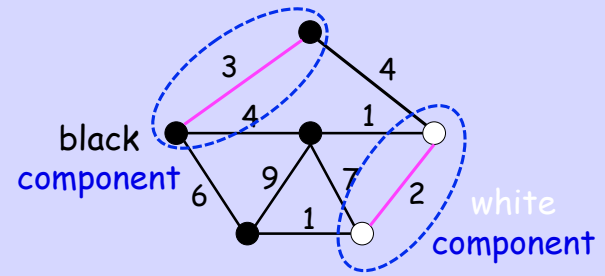


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mintree.13

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Minimum Spanning Trees re-color components

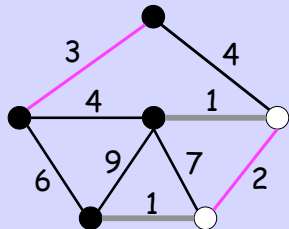


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mintree.14

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Minimum Spanning Trees gray edges

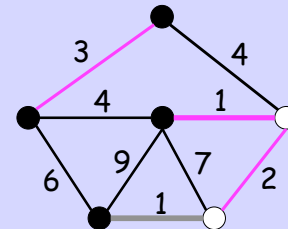


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mintree.15

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Minimum Spanning Trees gray edges: min weight

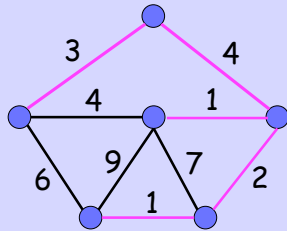


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mintree.16

6	9	13	7
12		10	5
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Minimum Spanning Trees



etc.



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mintree.18

6	9	13	7
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Ways to grow an MST

- start at any vertex, keep building one tree. (Prim)
- keep choosing min weight edge between diff components (Kruskal)
- grow trees in parallel (Boruvka)
- All special cases of gray edges



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mintree.19

6	9	13	7
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Enough Gray Edges

We have shown

Theorem: Any connected graph has a spanning tree whose edges are all min weight gray edges.

Still to prove:

any such tree is min-weight



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mintree.20