Minimum Weight Spanning Trees

Suppose edges have weights:

Find \text{min weight} spanning tree?

Build MST using \textit{gray} edges

- Start with vertices, no edges
- Color components black & white
- Graph is connected, so have:
  - gray edge ::= \[ \textcolor{gray}{\begin{array}{c} \text{gray edge} \\ \hline \end{array}} \]
  - add min weight gray edge

Minimum Spanning Trees

color components

initial components: isolated vertices
Minimum Spanning Trees

**color components**

```
4 1 3
1 2
6 9 7
```

**initial components:**
isolated vertices

Minimum Spanning Trees

**gray edges**

```
4 1 3
1 2
6 9 7
```

**gray edges: min weight**

```
4 1 3
1 2
6 9 7
```

re-color components

```
4 1 3
1 2
6 9 7
```

repeat:
Minimum Spanning Trees
re-color components

so gray edges must differ from previously selected

gray edges: min weight

repeat:
Minimum Spanning Trees

re-color components

gray edges

gray edges: min weight
Minimum Spanning Trees

Ways to grow an MST

- start at any vertex, keep building one tree. (Prim)
- keep choosing min weight edge between diff components (Kruskal)
- grow trees in parallel (Boruvka)
- All special cases of gray edges

Enough Gray Edges

We have shown

Theorem: Any connected graph has a spanning tree whose edges are all min weight gray edges.

Still to prove:
any such tree is min-weight