Mathematics for Computer Science

MIT 6.042J/18.062J

Structural Induction

Structural Induction

To prove P(x) holds for all x in recursively defined set R, prove

- P(b) for each base case $b \in R$
- P(c(x)) for each constructor, c, assuming ind. hyp. P(x)

$E \subseteq$ Even

by structural induction on $x \in E$ with ind. hyp. "x is even"

- 0 is even
- if n is even, then so is n+2, -n

Matched Paren Strings M

Lemma: Every s in M has the same number of]'s and ['s.

Proof by structural induction on the definition of M

Matched Paren Strings M

Lemma: Every s in M has the same number of]'s and [s.

Let EQ ::= {strings with same number of] and [}

Lemma (restated): $M \subseteq EQ$

Structural Induction on M

Proof:

Ind. Hyp. P(s) ::= ($s \in EQ$)

Base case ($s = \lambda$):

λ has 0]'s and 0 ['s,

so P(λ) is true.

base case is OK

Structural Induction on M

Constructor step: s = [r]t

can assume P(r) and P(t)

#] in s =	#] in r	+	#] in t	+ 1
#[in s =	#[in r	+	#[in t	+ 1
so =	= by P(r)		= by P(t)	

so P(s) is true constrct case is OK

Structural Induction on M

so by struct. induct.

$M \subseteq EQ$

QED

The 18.01 Functions, F18

Lemma.

F18 is closed under taking derivatives:

if $f \in F18$, then $f' \in F18$

Class Problem