Mathematics for Computer Science
MIT 6.042J/18.062J

Factorials: Stirling's Formula

Closed form for n!

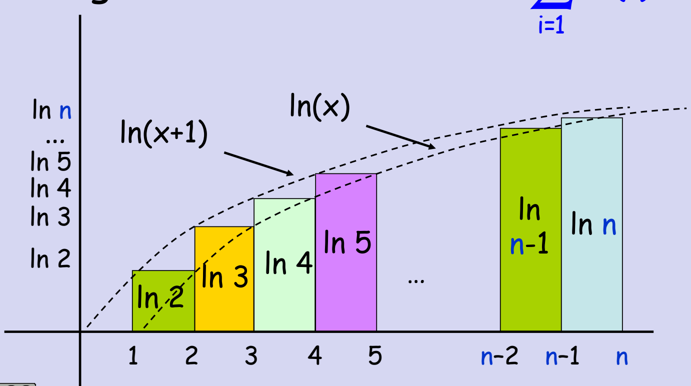
$$n! ::= 1\cdot 2\cdot 3\cdots(n-1)\cdot n = \prod_{i=1}^{n} i$$

Turn product into a sum taking logs:

$$\ln(n!) = \ln(1\cdot 2\cdot 3\cdots(n-1)\cdot n) =$$
$$\ln 1 + \ln 2 + \cdots + \ln(n-1) + \ln(n)$$
$$= \sum_{i=1}^{n} \ln(i)$$

Closed form for n!

Integral Method to bound $\sum_{i=1}^{n} \ln(i)$



Closed form for n!

$$n\ln\left(\frac{n}{e}\right)+1 \le \sum_{i=1}^{n} \ln(i) \le$$
$$(n+1)\ln\left(\frac{n+1}{e}\right)+0.6$$

reminder:

$$\int \ln x\, dx = x\ln\left(\frac{x}{e}\right)$$

Closed form for n!

$$\sum_{i=1}^{n} \ln(i) \approx (n+\frac{1}{2})\ln\left(\frac{n}{e}\right)$$

exponentiating:

$$n! \approx \sqrt{n/e}\left(\frac{n}{e}\right)^n$$

Stirling's Formula

A precise approximation:

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$