Mathematics for Computer Science

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Optimal Stable Matching

Mating Ritual

Who does better,
boys or girls?

Girls' suitors get better,
and boys' sweethearts
get worse

Mating Ritual

Who does better,
boys or girls?

Girls' suitors get better,
and boys' sweethearts
get worse, so girls do
better? No!

Feasible spouses

Nicole and Keith are
feasible spouses when
they are married in
some set of stable
marriages.

Optimal spouses

Nicole is optimal for Keith when she is Keith's highest ranked feasible spouse

Boys win the Ritual

In Mating Ritual:

Every boy marries his optimal wife.

Boys win the Ritual

Follows from Invariant

No girl has rejected a feasible spouse

Boys win the Ritual

Assume the Invariant.

Boy's wife is top-listed girl on Wedding Day.

Boys win the Ritual

Assume the Invariant.

Boy's wife is top-listed girl on Wedding Day. Deleted girls not feasible, so wife is highest ranked feasible.

optimal wife

Verify the Invariant

No girl has rejected a feasible spouse

Verify the Invariant

Suppose Invariant holds when Nicole rejects Bob. Must show that Bob is not feasible for Nicole.

Verify the Invariant

Nicole rejects Bob

…because Tom is serenading her and she prefers Tom to Bob.

Verify the Invariant

By Invariant, Tom's feasible wives still on his list.

Verify the Invariant

By Invariant, Tom's feasible wives still on his list. Nicole is top of list, so ranks above all Tom's other feasibles.

Verify the Invariant

So Nicole cannot stably marry Bob, because

Verify the Invariant

So Nicole cannot stably marry Bob, because she and Tom would be rogue:
She prefers Tom to Bob, and Tom would prefer her to whoever his wife is.

Verify the Invariant

So rejected suitor Bob is indeed not feasible:

Invariant is preserved!

Girls lose the Ritual

Easier argument shows each girl gets worst possible spouse.

Boys win, Girls lose

In Mating Ritual:

Every boy marries his optimal wife.

Every girl marries her pessimal husband.

Stable Marriage

More questions, rich theory

- other stable marriages possible? -can be many
- do better by lying? boys -No! girls -Yes!