Stable Matching

A Marriage Problem

1 2 3 4 5

Boys

Girls

A B C D E

Preferences

Boys

Girls

1: CBEAD
2: ABECED
3: DCBAE
4: ACDBE
5: ABDE

1: 35214
2: 52143
3: 43512
4: 12345
5: 23415

Try "greedy" strategy for boys
Stable Marriage

Preferences

1: CBED
2: ABED
3: DBAE
4: ADBE
5: ABDE

Marry Boy 1 with Girl C (his 1st choice)

Preferences

2: ABE D
3: D BAE
4: A DBE
5: ABDE

Stable Marriage

Preferences

2: ABED
3: DBAE
4: ADBE
5: ABDE

Next:

Marry Boy 2 with Girl A (his remaining 1st choice)
Stable Marriage

Final "boy greedy" marriages

1 C
2 A
3 D
4 B
5 E

Trouble!

1
C

Boy 4 likes Girl C better than his wife.

4 B

and vice-versa

4
C

B
Stable Marriage

a rogue couple

1 2 3 4

1 3 5 4

Stable Marriage Problem:
Marry everyone without any rogue couples!

Stable Marriage

Let’s try it!

Stable Marriage I.

3 A 5 B 4 C

1 D 2 E

all girls get 1st choice
Stable Marriage II

5 A
3 D
1 E

2 B
4 C

“boy optimal”

Stable Marriage

More than a puzzle:

• College Admissions
  (original Gale & Shapley paper, 1962)

• Matching Hospitals & Residents.

• Matching Dance Partners.