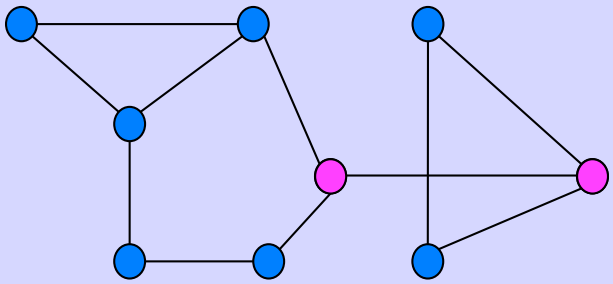
Mathematics for Computer Science
MIT 6.042J/18.062J

Simple Graphs: k-Connectivity

Edge Connectedness

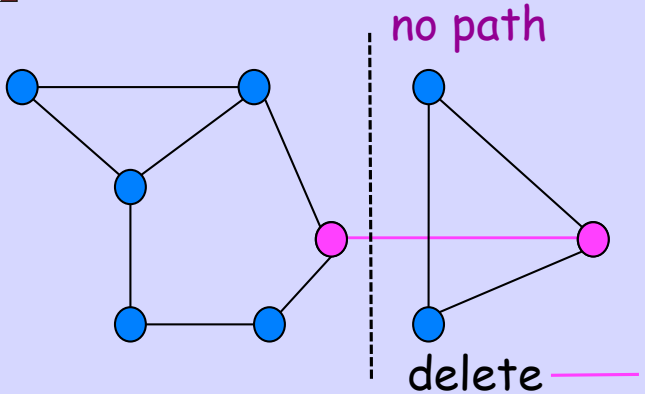
Def: vertices v, w are
k-edge connected
if they remain connected whenever fewer than k edges are deleted.

k-edge Connectedness



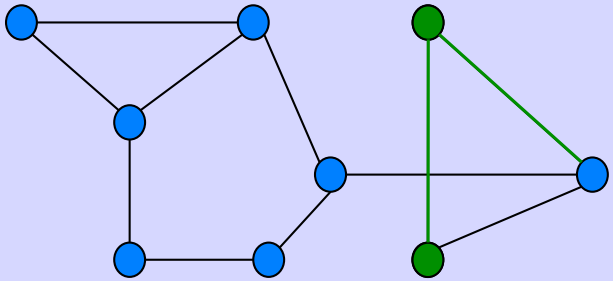
1-edge connected

k-edge Connectedness



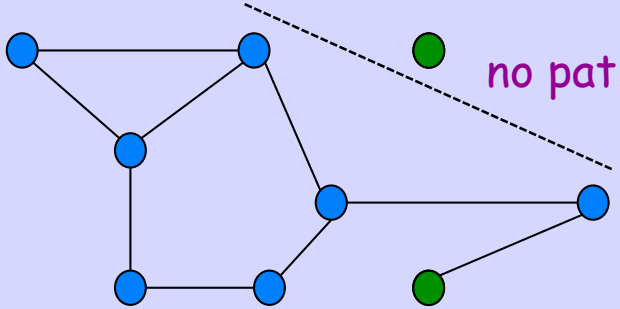
1-edge connected

Edge Connectedness



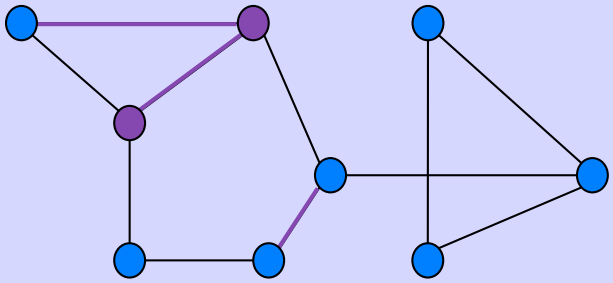
2-edge connected

Edge Connectedness



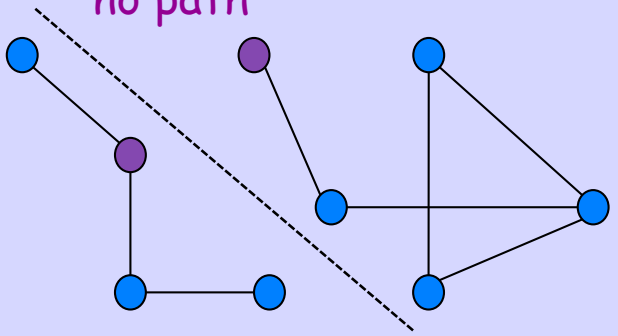
2-edge connected

Edge Connectedness



3-edge connected

Edge Connectedness



3-edge connected

k-edge Connectedness

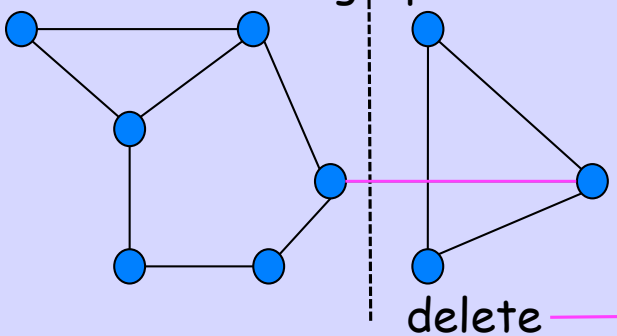
Def: A graph is
k-edge connected
iff every two vertices are k-edge connected.

Edge Connectedness

Connectivity measures fault tolerance of a network: how many connections can fail without cutting off communication?

k-edge Connectedness

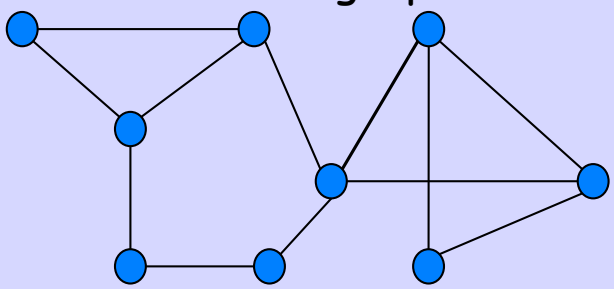
this whole graph is



1-edge connected

k-edge Connectedness

this whole graph is



2-edge connected

k-vertex Connectedness

k-vertex connectedness defined similarly

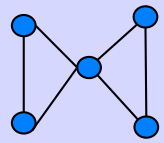
k-vertex Connectedness

k-vertex connected

IMPLIES

k-edge connected

not conversely:



2-edge connected

1-vertex connected

k-vertex Connectedness

K_n is the complete graph on n vertices.

K_n is (n-1)-vertex connected.

k-vertex Connectedness

The n-dimensional hypercube H_n

$V(H_n) ::= \{0,1\}^n$

$\langle u - v \rangle$ an edge IFF u,v differ in 1 place

k-vertex Connectedness

H_n is n-vertex connected.
(class problem)

Menger's Theorem

k-connected vertices will be connected by k non-overlapping paths