Simple Graphs: Isomorphism

The Graph Abstraction

Same graph (different layouts)

All that matters are the connections: graphs with the same connections are isomorphic

Same graph (different labels)
**Isomorphism**

Two graphs are **isomorphic** when there is an edge-preserving matching of their vertices.

**Are these isomorphic?**

- Dog → Beef
- Cat → Tuna
- Cow → Hay
- Pig → Corn

**Edges preserved?**

**YES!**
Nonedges preserved? **YES!**

![Graph Isomorphism Example](image1.png)

**Nonisomorphism**

- Degree 2
- All degree 3

**Formal Def of Graph Isomorphism**

$G_1$ **isomorphic** to $G_2$ means edge-preserving vertex matching:

\[
\exists \text{ bijection } f: V_1 \rightarrow V_2 \text{ with } u-v \text{ in } E_1 \iff f(u)-f(v) \text{ in } E_2
\]

**Proving nonisomorphism**

If some property **preserved by isomorphism** differs for two graphs, then they’re **not** isomorphic:

- # of nodes,
- # of edges,
- Degree distributions, ....
Finding an isomorphism?

many possible mappings: *large search*

can use properties *preserved* by

isomorphisms as a guide, for example:

• a deg 4 vertex adjacent to a deg 3
  can only match with

• a deg 4 vertex also adjacent to a deg 3

but even so...

Are these two graphs isomorphic?

...nothing known is *sure* to be

much faster than searching thru

all bijections for an isomorphism