

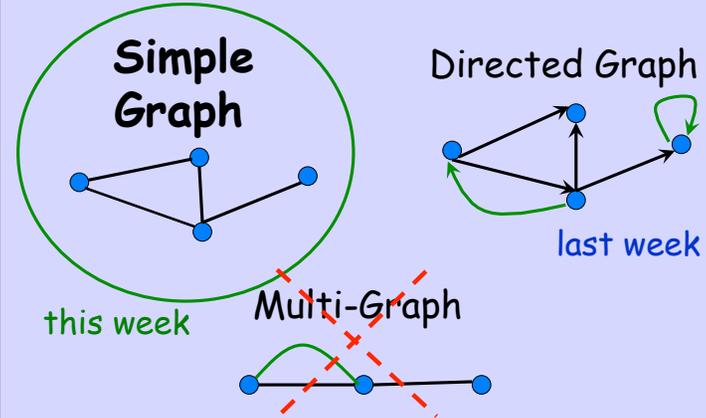
6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Simple Graphs: Degrees



6	9	13	7
12	10	5	
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Types of Graphs



6	9	13	7
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A simple graph:

Definition:

- A **simple graph** G consists of
- a nonempty set, V , of **vertices**, and
 - a set, E , of **edges** such that each edge has **two endpoints** in V



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A Simple Graph

vertices, V
undirected edges, E

edge

$$\text{red node} - \text{yellow node} ::= \{\text{red node}, \text{yellow node}\}$$

"adjacent"

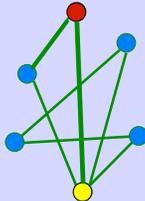


6	9	13	7
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Vertex degree

degree of a vertex is
of incident edges

$$\deg(\bullet) = 2$$



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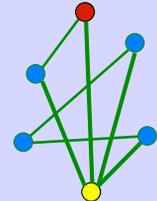
degrees.6

6	9	13	7
12	10	5	
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Vertex degree

degree of a vertex is
of incident edges

$$\deg(\bullet) = 4$$



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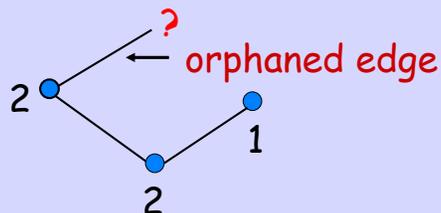
degrees.7

6	9	13	7
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Impossible Graph

Is there a graph with
vertex degrees 2,2,1?

NO!



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degrees.8

6	9	13	7
12	10	5	
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Handshaking Lemma

sum of degrees is
twice # edges

$$2|E| = \sum_{v \in V} \deg(v)$$

Proof: Each edge contributes
2 to the sum on the right



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degrees.9

6	9	13	7
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Handshaking Lemma

sum of degrees is
twice # edges

$$2|E| = \sum_{v \in V} \deg(v)$$

2+2+1 = odd,
so impossible



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degrees.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Sex in America: Men more Promiscuous?

- Univ Chicago, *The Social Organization of Sexuality*, 1994: men have **74%** more women partners than women have men
- NBC News, *American Sex Survey*, 2004: men have **233%** more women partners
- U.S. National Center for Health Statistics study, 2007: men have **175%** more women partners



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sexgraph.11

6	9	13	7
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Sex in America: Men more Promiscuous?

Studies claim different %'s
but agree that
**men average many more
partners than women.**

Graph theory shows
this is **nonsense**

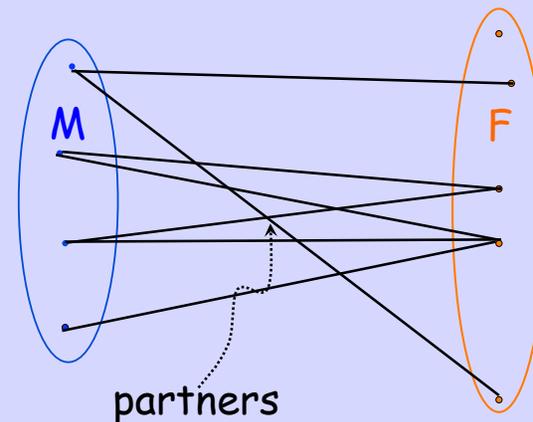


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sexgraph.12

6	9	13	7
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3	1	4	14
15	8	11	2

Sex Partner Graph



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degrees.13

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Counting pairs of partners

$$\text{avg degree}(M) ::= \frac{\sum_{m \in M} \text{deg}(m)}{|M|}$$

$$\text{avg degree}(F) ::= \frac{\sum_{f \in F} \text{deg}(f)}{|F|}$$



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degrees.14

6	9	13	7
12		10	5
3	1	4	14
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Counting pairs of partners

$$\frac{\text{avg degree}(M)}{\text{avg degree}(F)} = \frac{\sum_{m \in M} \text{deg}(m)}{\sum_{f \in F} \text{deg}(f)}$$

$$\sum_{m \in M} \text{deg}(m) = |E| = \sum_{f \in F} \text{deg}(f)$$



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degrees.16

6	9	13	7
12		10	5
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Counting pairs of partners

$$\frac{\text{avg degree}(M)}{\text{avg degree}(F)} = \frac{|F|}{|M|}$$



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degrees.17

6	9	13	7
12		10	5
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Average number of partners

$$\text{avg-deg}(M) = 1.035 \cdot \text{avg-deg}(F)$$

Averages differ solely by
ratio of females to males.

No big difference
Nothing to do with promiscuity



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degrees.19

6	9	13	7
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Why are surveys wrong?

Maybe people are **lying**:

- Males exaggerate?
- Females deny?

Maybe Males have partners
outside the study

