Simple Graphs: Coloring

flights need gates, but times overlap. how many gates needed?

Airline Schedule

Conflicts Among 3 Flights

Needs gate at same time
Model all Conflicts with a Graph

Color the vertices

Color vertices so that adjacent vertices have different colors.

\[ \text{min # distinct colors} = \text{min # gates needed} \]

Coloring the Vertices

Better coloring

Assign gates:

- 257, 67
- 122, 145
- 99
- 306

4 colors
4 gates

3 colors
3 gates
**Final Exams**

subjects **conflict** if student takes both, so need different time slots. how short an exam period?

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**Model as a Graph**

*Model* assigns times: 4 time slots (best possible)

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**Conflicting Allocation Problems**

# separate habitats to house different species of animals, some **incompatible** with others?
# different frequencies for radio stations that **interfere** with each other?
# different colors to **color** a map?

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**Map Coloring**

Map coloring.
Countries are the Vertices

Planar Four Coloring

any planar map is 4-colorable.
1850’s: false proof published (was correct for 5 colors).
1970’s: proof with computer
1990’s: much improved

Chromatic Number

\[
\text{min #colors for } G \text{ is chromatic number } \chi(G)
\]

Simple Cycles

\[
\chi(C_{\text{even}}) = 2
\]
\[
\chi(C_{\text{odd}}) = 3
\]
Complete Graph $K_5$

$\chi(K_n) = n$

The Wheel $W_n$

$\chi(W_{\text{even}}) = 3$
$\chi(W_{\text{odd}}) = 4$

Bounded Degree

all degrees $\leq k$, implies

$\chi(G) \leq k+1$

very simple algorithm...

“Greedy” Coloring

...color vertices in any order.
next vertex gets a color different from its neighbors.
$\leq k$ neighbors, so
$k+1$ colors always work
coloring arbitrary graphs

2-colorable? -- easy to check
3-colorable? -- hard to check
  (even if planar)

find $\chi(G)$? -- theoretically
  no harder than 3-color, but
  harder in practice