

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Sets: operations



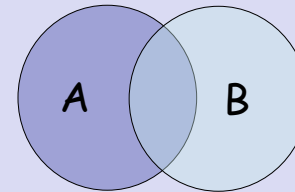
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sets-ops.1

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New sets from old



Venn Diagram for 2 Sets



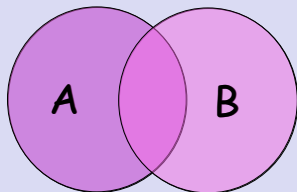
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sets-ops.2

6	9	13	7
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union



$$A \cup B ::= \{x \mid x \in A \text{ OR } x \in B\}$$



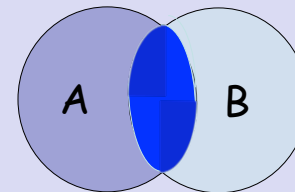
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sets-ops.3

6	9	13	7
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intersection



$$A \cap B ::= \{x \mid x \in A \text{ AND } x \in B\}$$



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sets-ops.4

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A set-theoretic equality

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

proof: Show these have the same elements, namely,
 $x \in \text{Left Hand Set}$ iff $x \in \text{RHS}$
 for all x .



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sets-ops.5

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A set-theoretic equality

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

proof uses fact from last time:

$$P \text{ OR } (Q \text{ AND } R) \text{ equiv} \\ (P \text{ OR } Q) \text{ AND } (P \text{ OR } R)$$



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sets-ops.6

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A set-theoretic equality

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

proof: $x \in A \cup (B \cap C)$ iff
 $x \in A$ OR $x \in (B \cap C)$ (def of \cup) iff
 $x \in A$ OR ($x \in B$ AND $x \in C$) (def \cap) iff
 $(x \in A$ OR $x \in B)$ AND $(x \in A$ OR $x \in C)$
 (by the equivalence)



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sets-ops.7

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A set-theoretic equality

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

proof: $x \in A \cup (B \cap C)$ iff
 $x \in A$ OR $x \in (B \cap C)$ (def of \cup) iff
 P OR (Q AND R) (def \cap) iff
 $(P$ OR $Q)$ AND $(P$ OR $R)$
 (by the equivalence)



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sets-ops.8

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A set-theoretic equality

proof:

$(x \in A \text{ OR } x \in B) \text{ AND } (x \in A \text{ OR } x \in C)$ iff
 $(x \in A \cup B) \text{ AND } (x \in A \cup C)$ (def \cup) iff
 $x \in (A \cup B) \cap (A \cup C)$ (def \cap).

QED



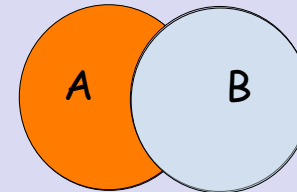
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sets-ops.9

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difference



$$A - B ::= \{x \mid x \in A \text{ AND } x \notin B\}$$



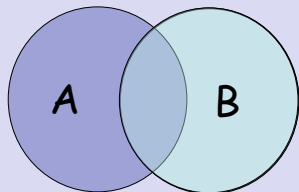
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sets-ops.10

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complement



$$\bar{A} ::= D - A = \{x \mid x \notin A\}$$



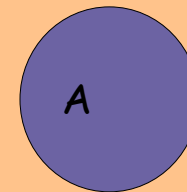
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sets-ops.12

6	9	13	7
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complement



$$\bar{A} ::= D - A = \{x \mid x \notin A\}$$



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sets-ops.13