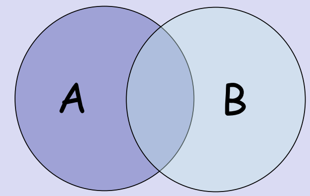
Mathematics for Computer Science

MIT 6.042J/18.062J

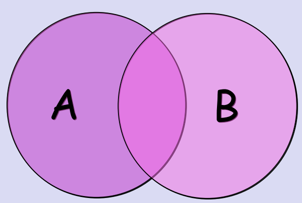
Sets: operations

New sets from old



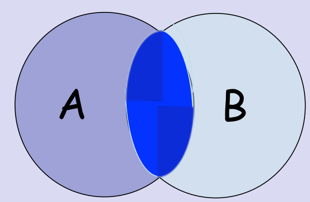
Venn Diagram for 2 Sets

union



$$A \cup B ::= \{x \mid x \in A \text{ OR } x \in B\}$$

intersection



$$A \cap B ::= \{x \mid x \in A \text{ AND } x \in B\}$$

A set-theoretic equality

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

proof: Show these have the same elements, namely,

$x \in$ Left Hand Set iff $x \in$ RHS

for all x.

A set-theoretic equality

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

proof uses fact from last time:

P OR (Q AND R) equiv

(P OR Q) AND (P OR R)

A set-theoretic equality

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

proof: $x \in A \cup (B \cap C)$ iff

$x \in A$ OR $x \in (B \cap C)$ (def of $\cup$) iff

$x \in A$ OR ($x \in B$ AND $x \in C$) (def $\cap$) iff

($x \in A$ OR $x \in B$) AND ($x \in A$ OR $x \in C$)

(by the equivalence)

A set-theoretic equality

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

proof: $x \in A \cup (B \cap C)$ iff

$x \in A$ OR $x \in (B \cap C)$ (def of $\cup$) iff

P OR (Q AND R) (def $\cap$) iff

(P OR Q) AND (P OR R)

(by the equivalence)

A set-theoretic equality

proof:

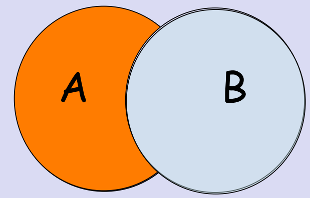
$(x \in A \text{ OR } x \in B) \text{ AND } (x \in A \text{ OR } x \in C)$ iff

$(x \in A \cup B) \text{ AND } (x \in A \cup C)$ (def $\cup$) iff

$x \in (A \cup B) \cap (A \cup C)$ (def $\cap$).

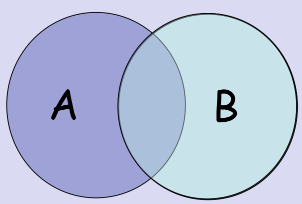
QED

difference



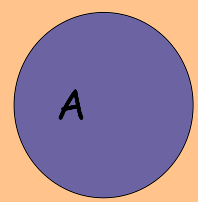
$$A - B ::= \{x \mid x \in A \text{ AND } x \notin B\}$$

complement



$$\overline{A} ::= D - A = \{x \mid x \notin A\}$$

complement



$$\overline{A} ::= D - A = \{x \mid x \notin A\}$$