Mathematics for Computer Science

MIT 6.042J/18.062J

Sets: definitions

What is a Set?

Informally:

A set is a collection of mathematical objects, with the collection treated as a single mathematical object.

(This is circular of course: what's a collection?)

Familiar sets

real numbers	$\mathbb{R}$
complex numbers	$\mathbb{C}$
integers	$\mathbb{Z}$
empty set	$\emptyset$

A set of 4 things

{7, "Albert R.", π/2, T}

A set with 4 elements: two numbers, a string, and a Boolean. Same as

{T, "Albert R.", 7, π/2}

-- order doesn't matter

In or Not In

An element is in or not in a set:

$\{7, \pi/2, 7\}$ is same as $\{7, \pi/2\}$

No notion of being in the set more than once.

Membership

x is a member of A: $x \in A$

$\pi/2 \in \{7,$ "Albert R.", $\pi/2,$ T$\}$

$14/2 \in$

$\pi/3 \notin$

Synonyms for Membership

$x \in A$

x is an element of A

x is in A

examples:

$7 \in \mathbb{Z}$, $2/3 \notin \mathbb{Z}$, $\mathbb{Z} \in \{\mathrm{T}, \mathbb{Z}, 7\}$

Subset ($\subseteq$)

$A \subseteq B$ A is a subset of B

A is contained in B

Every element of A is also an element of B:

$\forall x\, [x \in A$ IMPLIES $x \in B]$

Subset

examples:

$\mathbb{Z} \subseteq \mathbb{R}$, $\mathbb{R} \subseteq \mathbb{C}$, $\{3\} \subseteq \{5,7,3\}$

$A \subseteq A$, $\varnothing \subseteq$ every set

$\varnothing \subseteq$ everything

$\varnothing \subseteq B$ is defined to mean

$$\forall x\ \underbrace{[\underbrace{x \in \varnothing}_{\text{false}} \text{ IMPLIES } x \in B]}_{\text{true}}$$

Defining Sets

The set of elements x in A

such that P(x) is true.

$$\{x \in A \mid P(x)\}$$

Defining Sets

The set of elements x in A

such that P(x) is true.

$$\{x \in A \mid P(x)\}$$

Defining Sets

The set E of even integers:

$$\left\{ n \in \mathbb{Z} \mid n \text{ is even} \right\}$$

Power Set

pow(A) ::= all the subsets of A

$= \{B \mid B \subseteq A\}$

example:

$\text{pow}(\{T, F\}) = \{\{T\}, \{F\}, \{T, F\}, \emptyset\}$

$E \in \text{pow}(\mathbb{Z})$, $\mathbb{Z} \in \text{pow}(\mathbb{R})$

$B \in \text{pow}(A)$ IFF $B \subseteq A$