

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mathematics for Computer Science
MIT 6.042J/18.062J

Representing Partial Orders

Albert R Meyer March 22, 2013 rep-po.1

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

proper subset relation

$A \subset B$ means
 B has everything
that A has
and more: $B \not\subset A$

Albert R Meyer March 22, 2013 rep-po.2

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

proper subset relation

Albert R Meyer March 22, 2013 rep-po.3

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |


partial order: properly divides

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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

same shape

as \subset example

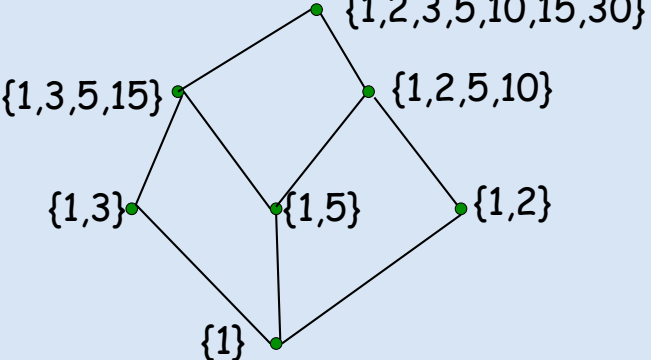


Albert R Meyer March 22, 2013

rep-po.5

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

proper subset

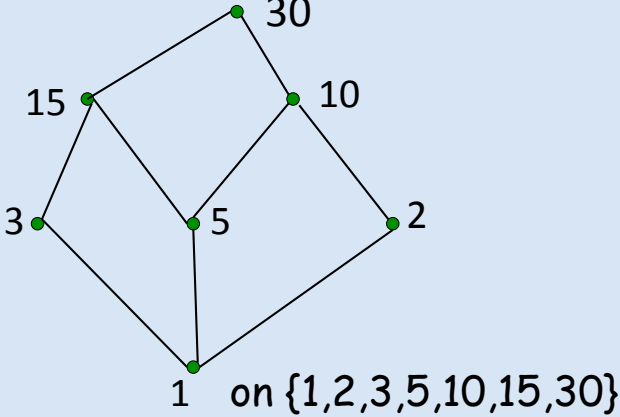


Albert R Meyer March 22, 2013


rep-po.6

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

partial order: properly divides



on $\{1,2,3,5,10,15,30\}$



Albert R Meyer March 22, 2013


rep-po.7

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

same shape

as \subset example

isomorphic



Albert R Meyer March 22, 2013

rep-po.8

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Isomorphism

All that matters
are the **connections**:
graphs with the
same connections
are **isomorphic**



Albert R Meyer March 19, 2012

rep-po.9

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Isomorphism

two graphs are **isomorphic**
when there is an
edge-preserving
bijection
of their vertices.



Albert R Meyer March 22, 2013

rep-po.10

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Formal Def of Graph Isomorphism

G_1 isomorphic to G_2 iff

\exists bijection $f: V_1 \rightarrow V_2$ with
 $u \rightarrow v$ in E_1 IFF $f(u) \rightarrow f(v)$ in E_2



Albert R Meyer March 19, 2012

rep-po.11

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

p.o. represented by \subset

Theorem: Every strict
partial order is isomorphic
to a collection of subsets
partially ordered by \subset .



Albert R Meyer March 22, 2013


rep-po.12

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

p.o. isomorphic to \subset

proof: map element, a , to the set of elements below it.

a maps to $\{b \in A \mid bRa \text{ OR } b=a\}$


 Albert R Meyer March 22, 2013 rep-po.13


| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

p.o. isomorphic to \subset

proof: map element, a , to the set of elements below it.

a maps to $\{b \in A \mid bRa \text{ OR } b=a\}$


$$f(a) ::= R^{-1}(a) \cup \{a\}$$


 Albert R Meyer March 22, 2013 rep-po.14

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

subsets from divides

$1 \rightarrow \{1\}$
 $2 \rightarrow \{1, 2\}$
 $3 \rightarrow \{1, 3\}$
 $5 \rightarrow \{1, 5\}$
 $6 \rightarrow \{1, 2, 3, 6\}$
 $10 \rightarrow \{1, 2, 5, 10\}$
 $15 \rightarrow \{1, 3, 5, 15\}$
 $30 \rightarrow \{1, 2, 3, 5, 10, 15, 30\}$


 Albert R Meyer March 22, 2013 rep-po.15