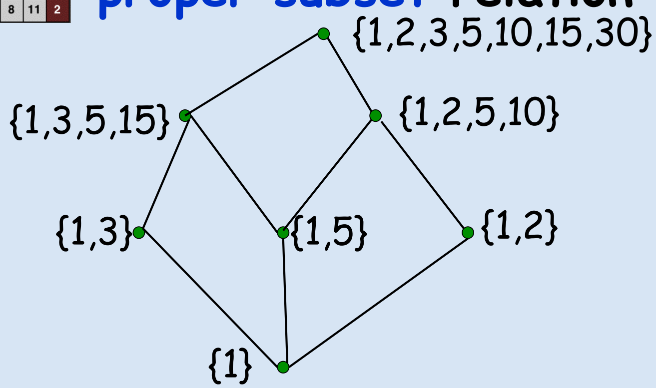
Mathematics for Computer Science
MIT 6.042J/18.062J

Representing Partial Orders

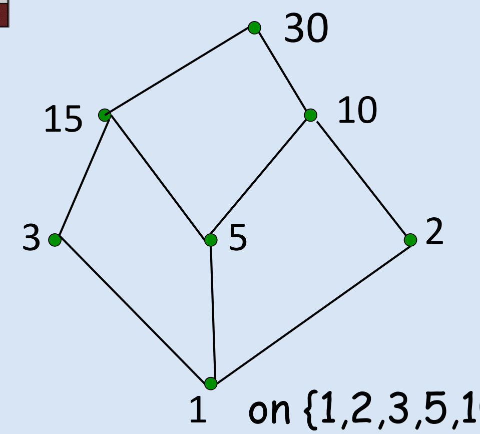
proper subset relation

$A \subset B$ means
B has everything
that A has
and more: $B \not\subset A$

proper subset relation



partial order: properly divides

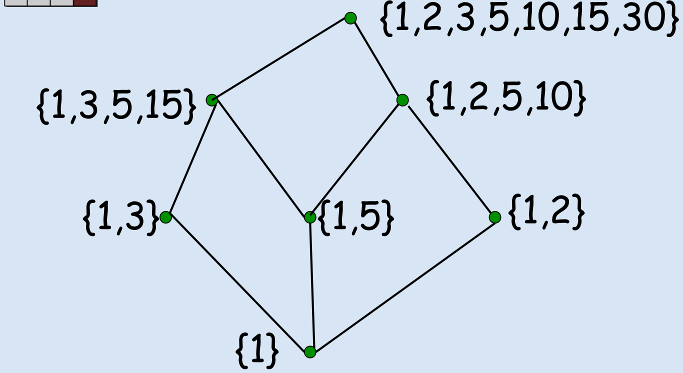


on {1,2,3,5,10,15,30}

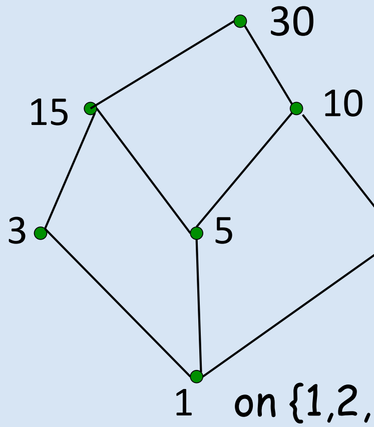
same shape

as ⊂ example

proper subset



partial order: properly divides



on {1,2,3,5,10,15,30}

same shape

as ⊂ example

isomorphic

Isomorphism

All that matters
are the connections:
graphs with the
same connections
are isomorphic

Isomorphism

two graphs are isomorphic
when there is an
edge-preserving
bijection
of their vertices.

Formal Def of Graph Isomorphism

G_1 isomorphic to G_2 iff

$\exists$ bijection $f{:}V_1 \rightarrow V_2$ with
$u \rightarrow v$ in E_1 IFF $f(u) \rightarrow f(v)$ in E_2

p.o. represented by $\subset$

Theorem: Every strict partial order is isomorphic to a collection of subsets partially ordered by $\subset$.

p.o. isomorphic to $\subset$

proof: map element, a, to the set of elements below it.

a maps to $\{b \in A \mid b\,R\,a \text{ OR } b = a\}$

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$$f(a) ::= R^{-1}(a) \cup \{a\}$$

subsets from divides