*Mathematics for Computer Science*

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# Recursive Data



## Recursive Definitions

Define something in terms of a simpler version of the same thing:

Base case(s) that don't depend on anything else.

Constructor case(s) that depend on simpler cases.



## Example Definition: set E

Define set $E \subseteq \mathbb{Z}$, recursively:

- Base case: $0 \in E$
- Constructor cases:

  If $n \in E$, then

  1. $n + 2 \in E$, if $n \geq 0$;
  2. $-n \in E$, if $n > 0$.



## Example Definition: set E

1. $n \in E$ and $n \geq 0$, then $n + 2 \in E$:

   0, 0+2, (0+2)+2, ((0+2)+2) +2

   0, 2, 4, 6, …

2. $n \in E$ and $n > 0$, then $-n \in E$

   -2, -4, -6, …

all even numbers

## Recursive Def: Extremal Clause

So, E contains the even integers

Anything Else? No!



## Recursive Def: Extremal Clause

So, E contains the even integers

- $0 \in E$
- If $n \in E$ and $n \geq 0$ , then $n+2 \in E$
- If $n \in E$ and $n > 0$, then $-n \in E$
- That's All!

Extremal Clause

(Implicit part of definition)



## Example Definition: set E

So E is exactly the Even Integers



## Matched Paren Strings, M

set of strings, $M \subseteq \{ \, ], [ \, \}^*$

- Base: $\lambda \in M$,

  (the empty string)

- Constructor:

  If $s, t \in M$, then

  $[s]t \in M$

## Matched Paren Strings M

strings $[s]t \in M$

| | | |
|---|---|---|
| [] | s = λ | t = λ |
| [[]] | s = [] | t = λ |
| [][] | s = λ | t = [] |
| [[]][] | s = [] | t = [] |
| [[[]]] | s = [[]] | t = λ |
| ⋮ | ⋮ | ⋮ |



## not in M

strings starting with ]

are not in M because

- λ does not start with ]
- [s]t does not start with ]

and everything in M arises in one of these two ways



## The 18.01 Functions, F18

The set F18 of functions on $\mathbb{R}$:

$Id_{\mathbb{R}}$, constant functions, and sin x are in F18.

if $f, g \in$ F18, then

- $f + g$, $f \cdot g$, $2^f$,
- the inverse, $f^{(-1)}$, of f, and
- $f \circ g$ (the composition of f and g)

are in F18.



## The 18.01 Functions, F18

Some functions in F18:

$-x = (-1) \cdot x$

$\sqrt{x} = (x^2)^{(-1)}$ ---inverse

$\cos x = (1 - (\sin x \cdot \sin x))^{1/2}$

$\ln x = (2^{x \log e})^{(-1)}$