Expected Number of Heads

Expected #Heads

n independent flips of a coin with bias p for Heads. How many Heads expected?

\[ E[\# \text{Heads}] = E[B_{n,p}] \]

\[ E[B_{n,p}] := \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} \]

\[ E[B_{n,p}] := \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} \]
Expected #Heads

Binomial theorem and differentiating gives a closed formula

\[(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}\]

take \( \frac{\partial}{\partial x} \):

\[n(x + y)^{n-1} = \frac{1}{x} \sum_{k=0}^{n} k \binom{n}{k} x^{k-1} y^{n-k}\]

Binomial Expectation

\[E\left[B_{n,p}\right] := \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k}\]

\[n(p + q)^{n-1} = \frac{1}{p} \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k}\]
Binomial Expectation

\[ E[B_{n,p}] := \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} \]

\[ n = \frac{1}{p} \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} \]

\[ np = E[B_{n,p}] \]