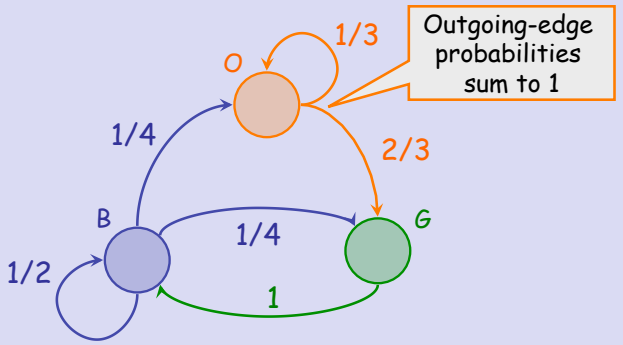
Mathematics for Computer Science

MIT 6.042J/18.062J

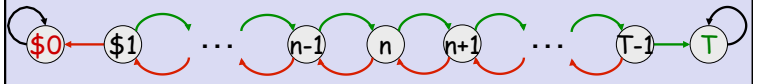
Random Walks

Graph With Probable Transitions

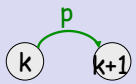


Example: Gambler's Ruin

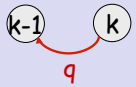
View as random walk on a line.



p ::=Pr[win a bet]



q ::= 1-p = Pr[lose a bet]

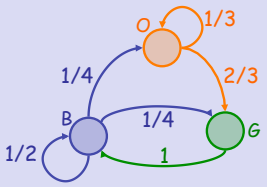


What is Pr[reach T before 0]?

Applications of Random Walk

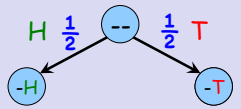
- Physics — Brownian motion
- Finance — stocks, options
- Algorithms — web search, clustering

Questions



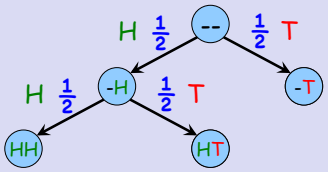
- Pr[reach O in 7 steps| start at B]
- Average # steps from B to O
- Pr[reach G before O | start at B]

Example: Toss HTH before TTH

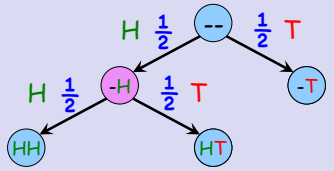


$$\Pr[\text{win}] = \Pr[\text{win}|\ \text{--}\ ]$$
$$= \tfrac{1}{2}\Pr[\text{win}|\ \text{-H}\ ] + \tfrac{1}{2}\Pr[\text{win}|\ \text{-T}\ ]$$

Example: Toss HTH before TTH

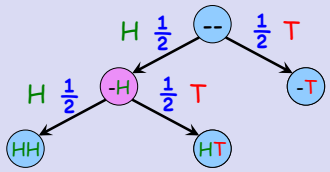


Example: Toss HTH before TTH



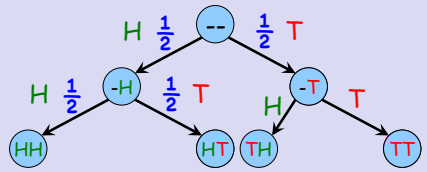
$$\Pr[\text{win}|\ \text{-H}\ ]$$
$$= \tfrac{1}{2}\Pr[\text{win}|\ \text{HH}\ ]$$

Example: Toss HTH before TTH

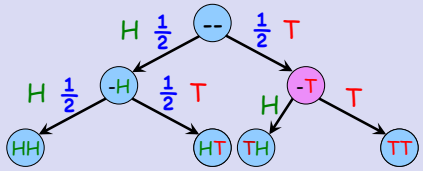


$$\Pr[\text{win}|\,(-H)\,] = \tfrac{1}{2}\Pr[\text{win}|\,(HH)\,] + \tfrac{1}{2}\Pr[\text{win}|\,(HT)\,]$$

Example: Toss HTH before TTH

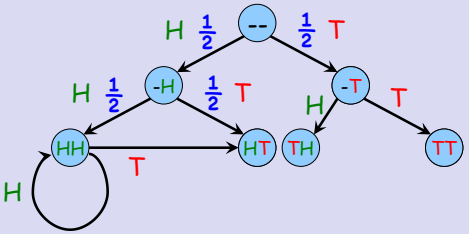


Example: Toss HTH before TTH

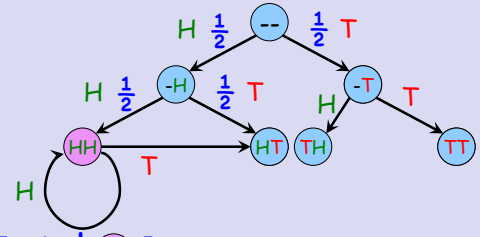


$$\Pr[\text{win}|\,(-T)\,] = \tfrac{1}{2}\Pr[\text{win}|\,(TH)\,] + \tfrac{1}{2}\Pr[\text{win}|\,(TT)\,]$$

Example: Toss HTH before TTH

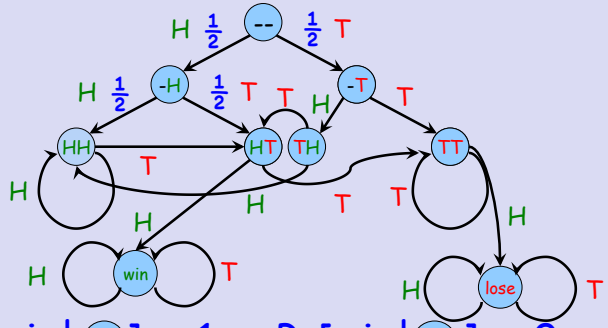


Example: Toss HTH before TTH



$$\Pr[\text{win}|\,HH] = \tfrac{1}{2}\Pr[\text{win}|\,HH] + \tfrac{1}{2}\Pr[\text{win}|\,HT]$$

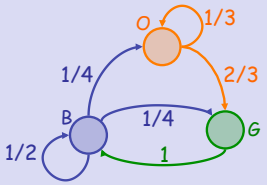
Example: Toss HTH before TTH



$\Pr[\text{win}|\,win] = 1$ $\Pr[\text{win}|\,lose] = 0$

Now solve system of linear equations for Pr[win]

Questions



- Pr[reach O in 7 steps| start at B]
- Average # steps from B to O
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Just solve systems of linear equations