

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Monty Hall Conditional Probability



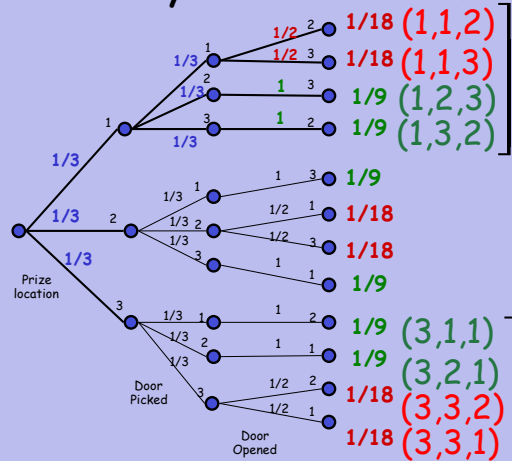
6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Monty Hall Conditional Probability often confusing



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Monty Hall Probabilities

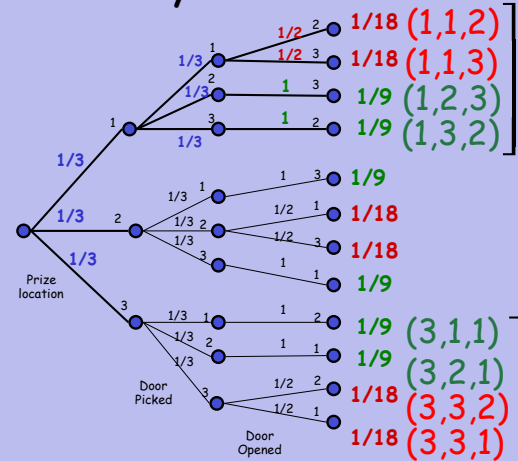


goat
at 2



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Monty Hall Probabilities



prize
at 1

goat
at 2



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Conditional Probability: Monty Hall

$\Pr[\text{prize at 1} \mid \text{goat at 2}]$

$$= \frac{1}{2} \quad \text{Really!}$$

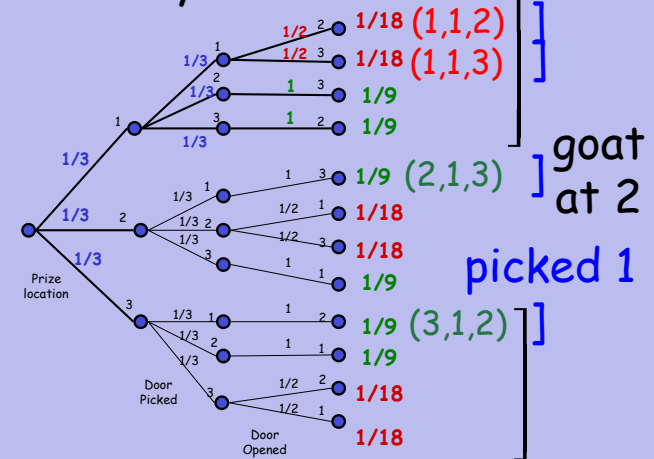


Albert R Meyer, May 3, 2013

condmonty.5

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Monty Hall Probabilities

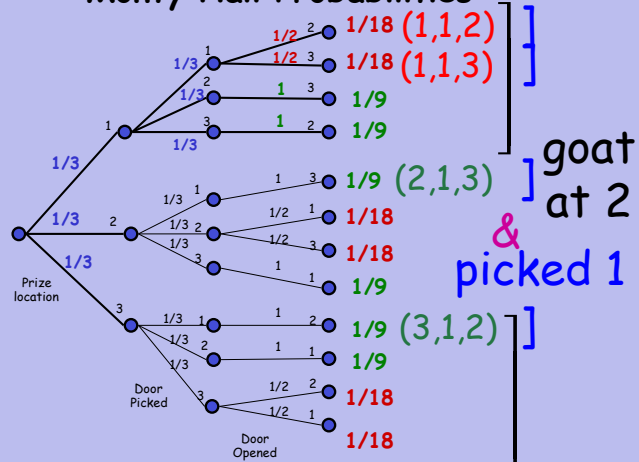


Albert R Meyer, May 3, 2013

condmonty.13

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Monty Hall Probabilities

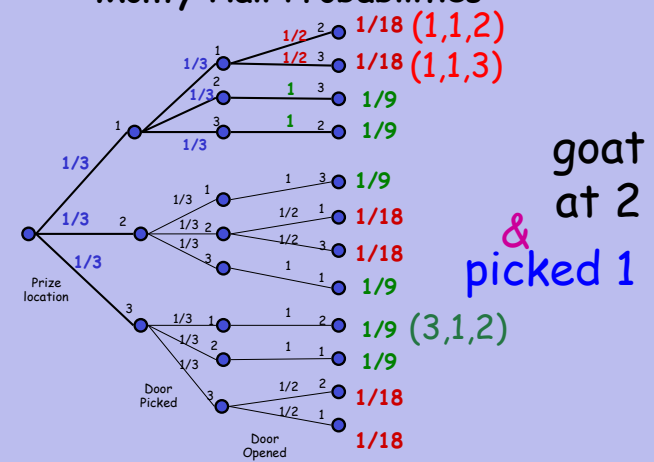


Albert R Meyer, May 3, 2013

condmonty.14

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Monty Hall Probabilities



Albert R Meyer, May 3, 2013

condmonty.15

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Conditional Probability: Monty Hall

Pr[prize at 1 | picked 1 & goat at 2]

$$= \frac{1}{2} \quad \text{Also!}$$



Albert R Meyer,

May 3, 2013

condmonty.16

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stick or Switch?

Seems the contestant may as well **stick**, since the probability is $1/2$ given **what he knows** when he chooses. **Wait!** contestant **knows more** than **what door he picked & where a goat is**, he knows

what door Carol opened!



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May 3, 2013

condmonty.21

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Conditional Probability: Monty Hall

So until now, we have been **conditioning on the wrong events** — a common blunder.

Using the **correct one**:



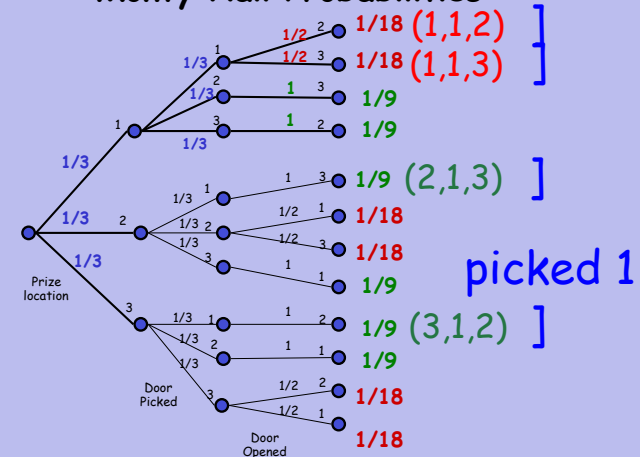
Albert R Meyer,

May 3, 2013

condmonty.22

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

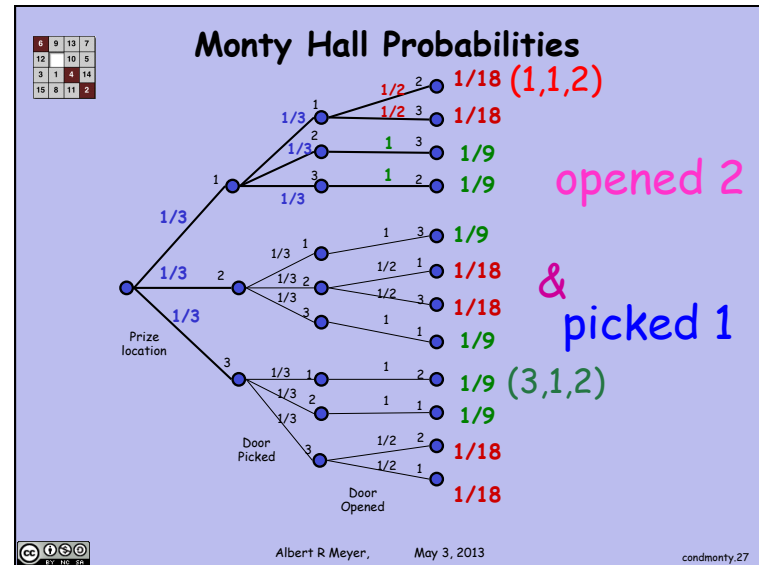
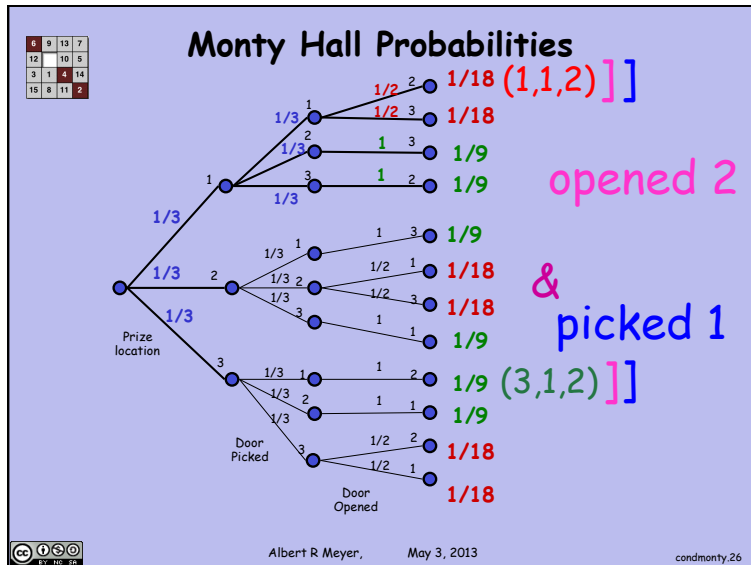
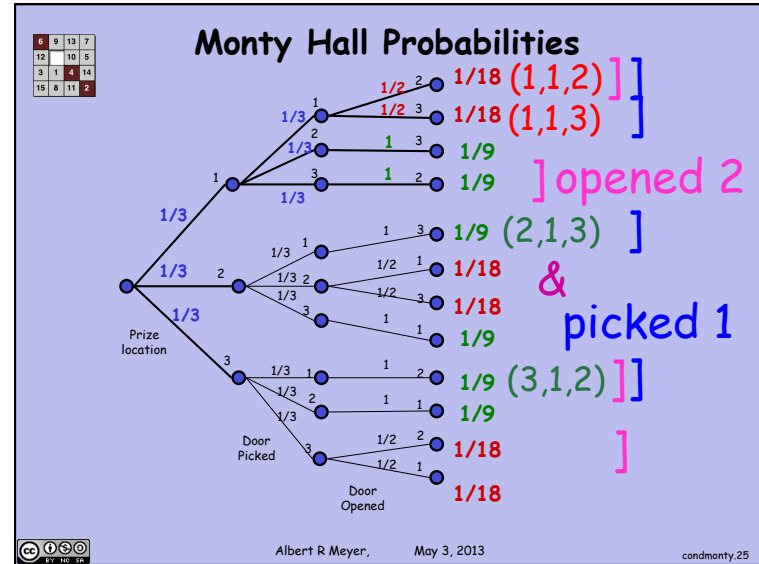
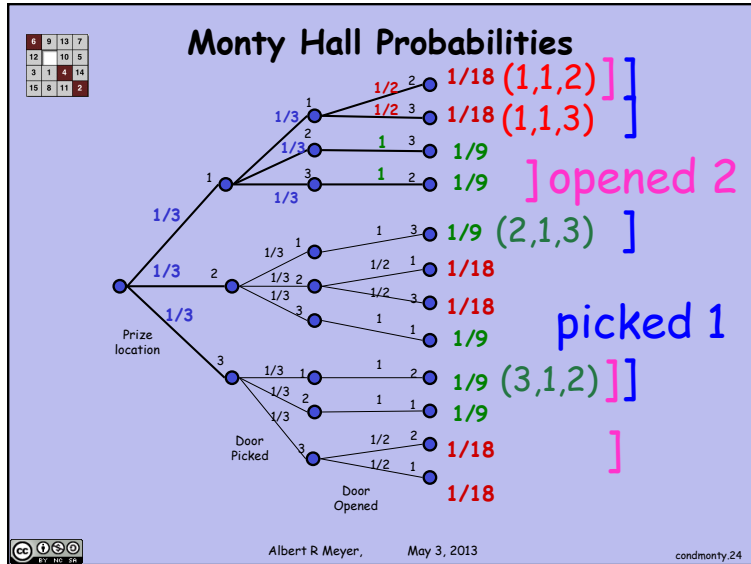
Monty Hall Probabilities



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May 3, 2013

condmonty.23



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Conditional Probability: Monty Hall

prize at 1

$$\text{Pr}[\text{picked 1 \& opened 2}] = \{(1,1,2)(3,1,2)\}$$



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condmonty.29

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Conditional Probability: Monty Hall

$\text{Pr}[\text{prize at 1} \mid \text{picked 1 \& opened 2}]$

$$\text{Pr}[\text{picked 1 \& opened 2}] = \{(1,1,2)(3,1,2)\}$$

$\underbrace{\hspace{1.5cm}}_{\text{Pr}=1/18} \quad \underbrace{\hspace{1.5cm}}_{\text{Pr}=1/9}$



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condmonty.30

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Conditional Probability: Monty Hall

$\text{Pr}[\text{prize at 1} \mid \text{picked 1 \& opened 2}]$

$$= \frac{1/18}{1/18 + 1/9}$$



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condmonty.31

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Conditional Probability: Monty Hall

$\text{Pr}[\text{prize at 1} \mid \text{picked 1 \& opened 2}]$

$$= \frac{1/18}{1/18 + 1/9} = \frac{1}{3}$$

$$= \text{Pr}[\text{sticking wins}]$$



Albert R Meyer, May 3, 2013

condmonty.32

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stick or Switch?

$$\begin{aligned} & \Pr[\text{prize at 1} \mid \text{picked 1 \& opened 2}] \\ &= \frac{1/18}{1/18 + 1/9} = \frac{1}{3} \\ &= \Pr[\text{sticking wins}] \end{aligned}$$



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condmonty.33

6	9	13	7
12	10	5	
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15	8	11	2

Switch!

By conditioning on **everything the contestant knows**, we've finally confirmed what we learned earlier:

$$\Pr[\text{switching wins}] = \frac{2}{3}$$



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condmonty.34

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

The 4 Step Method

It's easy to see how so many smart people get confused by Monty Hall.



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condmonty.35

6	9	13	7
12	10	5	
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15	8	11	2

The 4 Step Method

It's easy to see how so many smart people get confused by Monty Hall. Finding the right event to condition on can be tricky.



Albert R Meyer, May 3, 2013

condmonty.36

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

The 4 Step Method

It's easy to see how so many smart people get confused by Monty Hall. Finding the right event to condition on can be tricky. The 4 step method is a good fall back approach.



Albert R Meyer,

May 3, 2013

condmonty.37