## Mathematics for Computer Science MIT 6.042J/18.062J <br> Law of Total Probability



Albert R Meyer, May 3, 2013

Law of Total Probability
Law for reasoning about probability by cases

$$
\begin{aligned}
& \text { Law of Total Probability } \\
& A=\left(B_{1} \cap A\right) \cup\left(B_{2} \cap A\right) \cup\left(B_{3} \cap A\right) \\
& \operatorname{Pr}[A]= \operatorname{Pr}\left[A \mid B_{1}\right] \operatorname{Pr}\left[B_{1}\right] \\
& \operatorname{Pr}\left[B_{2} \cap A\right]+ \\
& \operatorname{Pr}\left[B_{3} \cap A\right]
\end{aligned}
$$

$$
\begin{aligned}
& \text { Law of Total Probability } \\
& \text { If } \mathcal{S} \text { is disjoint union of } B_{0}, B_{1}, \ldots \\
& \operatorname{Pr}[A]=\sum_{i} \operatorname{Pr}\left[A \cap B_{i}\right] \\
& =\sum_{i} \operatorname{Pr}\left[A \mid B_{i}\right] \cdot \operatorname{Pr}\left[B_{i}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \text { Law of Total Probability } \\
& \operatorname{Pr}[A]=\operatorname{Pr}\left[A \mid B_{1} \cap \operatorname{A}\right)\left(B_{3} \cap A\right) \\
& +\operatorname{Pr}\left[A \mid B_{1}\right] \\
& \quad+\operatorname{Pr}\left[A \mid B_{2}\right] \operatorname{Pr}\left[B_{2}\right] \\
&
\end{aligned}
$$

