Law of Total Probability

Law for reasoning about probability by cases

\[ A = (B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A) \]

\[ \Pr[A] = \Pr[B_1 \cap A] + \Pr[B_2 \cap A] + \Pr[B_3 \cap A] \]
Law of Total Probability

\[ A = (B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A) \]

\[ \Pr[A] = \Pr[A|B_1] \Pr[B_1] \]
\[ + \Pr[B_2 \cap A] \]
\[ + \Pr[B_3 \cap A] \]

Law of Total Probability

\[ A = (B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A) \]

\[ \Pr[A] = \Pr[A|B_1] \Pr[B_1] \]
\[ + \Pr[A|B_2] \Pr[B_2] \]
\[ + \Pr[A|B_3] \Pr[B_3] \]

Law of Total Probability

If \( S \) is disjoint union of \( B_0, B_1, \ldots \)

\[ \Pr[A] = \sum_i \Pr[A \cap B_i] \]
\[ = \sum_i \Pr[A|B_i] \cdot \Pr[B_i] \]