Mathematics for Computer Science
MIT 6.042J/18.062J

Law of Total Probability

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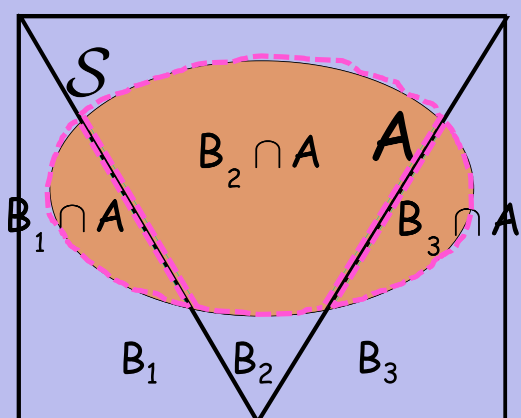
Law for reasoning about probability by cases

Law of Total Probability



Law of Total Probability

$$A = (B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A)$$

$$\Pr[A] = \Pr[B_1 \cap A] + \Pr[B_2 \cap A] + \Pr[B_3 \cap A]$$

Law of Total Probability

$$A = (B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A)$$

$$\Pr[A] = \Pr[A \mid B_1]\,\Pr[B_1]$$
$$\Pr[B_2 \cap A] +$$
$$\Pr[B_3 \cap A]$$

Law of Total Probability

$$A = (B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A)$$

$$\Pr[A] = \Pr[A \mid B_1]\,\Pr[B_1] + \Pr[A \mid B_2]\,\Pr[B_2] + \Pr[A \mid B_3]\,\Pr[B_3]$$

Law of Total Probability

If $\mathcal{S}$ is disjoint union of $B_0, B_1, \ldots$

$$\Pr[A] = \sum_i \Pr[A \cap B_i]$$
$$= \sum_i \Pr[A \mid B_i] \cdot \Pr[B_i]$$