

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
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Mutually Independent Events



Albert R Meyer, May 3, 2013

mutualindep.1

6	9	13	7
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Mutual Independence

Events A_1, A_2, \dots, A_n are mutually independent when the probability that A_i occurs



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mutualindep.2

6	9	13	7
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Mutual Independence

Events A_1, A_2, \dots, A_n are mutually independent when the probability that A_i occurs is unchanged by which other ones occur.



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mutualindep.3

6	9	13	7
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Mutual Independence

Example: Successive coin flips
 $H_i ::= [i^{\text{th}} \text{ flip is Heads}]$
What happens on the 5th flip is independent of what happens on the 1st, 4th, or 7th flip:

$$\Pr[H_5] = \Pr[H_5 \mid H_1 \cap H_4 \cap \overline{H_7}]$$



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mutualindep.5

6	9	13	7
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Mutual Independence

Events A_1, A_2, \dots, A_n are mutually independent when

when

$$\Pr[A_i] = \Pr[A_i | A_j \cap A_k \cap \dots \cap A_m]$$

$(i \neq j, k, \dots, m)$



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mutualindep.7

6	9	13	7
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Mutual Independence

Events A_1, A_2, \dots, A_n are mutually independent when

when

$$\Pr[A_i \cap A_j \cap \dots \cap A_m] = \Pr[A_i] \cdot \Pr[A_j] \cdot \dots \cdot \Pr[A_m]$$



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mutualindep.8

6	9	13	7
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Pairwise Independence

Example: Flip a **fair** coin twice

$H_1 ::=$ [Head on 1st flip]

$H_2 ::=$ [Head on 2nd flip]

$O ::=$ [Odd # Heads]

Claim: O is independent of H_1



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Pairwise Independence

Example: Flip a **fair** coin twice

O is independent of H_1 :

$O = \{HT, TH\}$, $\Pr[O] = 1/2$

$O \cap H_1 = \{HT\}$, $\Pr[\{HT\}] = 1/4$

$\Pr[O \cap H_1] = 1/4 = \Pr[O] \cdot \Pr[H_1]$



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mutualindep.11

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Not Mutually Independent

Example: Flip a fair coin twice

But O, H_1, H_2 not mutually independent:

$$\Pr[O | H_1 \cap H_2] = 0 \neq \Pr[O]$$



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mutualindep.12

6	9	13	7
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k-way Independence

Example: Flip a fair coin k times

$H_i ::=$ [Head on i^{th} flip]

$O ::=$ [Odd # Heads]

Claim: Any set of k of these events are mutually independent, but all $k+1$ of them are not.



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mutualindep.14

6	9	13	7
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k-way Independence

Events A_1, A_2, \dots are

k-way independent

iff any k of them are mutually independent.

Pairwise = 2-way



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mutualindep.15

6	9	13	7
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k-way Independence

Events A_1, A_2, \dots are

k-way independent

iff any k of them are mutually independent.

O, H_1, \dots, H_k are k-way, not $(k+1)$ -way independent



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mutualindep.16

6	9	13	7
12	10	5	
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Mutual Independence

Events A_1, A_2, \dots, A_n are
mutually independent
when they are n -way independent

$\left(2^n - (n+1) \text{ equations} \right)$
to check!



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mutualindep.17