Mathematics for Computer Science
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Probability Spaces

Probability Spaces

1) Sample space: a countable set $\mathcal{S}$ whose elements are called outcomes
2) Probability function, Pr: $\mathcal{S} \rightarrow [0, 1]$, such that

$$\sum_{\omega \in \mathcal{S}} \Pr[\omega] = 1$$

Probability Spaces

The purpose of the "tree model" is to figure out which probability space to use:

- outcomes = leaves of tree
- outcome probabilities calculated from branch probabilities.

Probability Spaces

An event is a subset, $E \subseteq \mathcal{S}$.

$$\Pr[E] ::= \sum_{\omega \in E} \Pr[\omega]$$

Cor: The Sum Rule

Sum Rule

For pairwise disjoint $A_0, A_1, \ldots$

$$\Pr[A_0 \cup A_1 \cup A_2 \cup \cdots] = \Pr[A_0] + \Pr[A_1] + \Pr[A_2] + \cdots$$

Sum Rule

For pairwise disjoint $A_0, A_1, \ldots$

$$\Pr\left[\bigcup_{i \in \mathbb{N}} A_i\right] = \sum_{i \in \mathbb{N}} \Pr[A_i]$$

Discrete Probability

Discrete = countable sample space

Allows sums instead of integrals