Mathematics for Computer Science
MIT 6.042J/18.062J

Predicate Logic, I
Quantifiers ∀,∃

Predicates

Propositions with variables

Example:

$P(x,y) ::= [x + 2 = y]$

Predicates

$P(x,y) ::= [x + 2 = y]$

$x = 1$ and $y = 3$: $P(1,3)$ is true

$x = 1$ and $y = 4$: $P(1,4)$ is false

NOT($P(1,4)$) is true

Quantifiers

$\forall x$ For ALL x

$\exists y$ There EXISTS some y

∀ is like AND

Let s range over 6.042 staff

P(s) ::= [s is Pumped about 6.042]

$\forall s.\ P(s)$

same as

P(Drew) AND P(Peter) AND P(Keshav) AND…AND P(Michaela)

∃ is like OR

Let t range over 6.042 staff

B(t) ::= [t took 6.042 Before]

$\exists t.\ B(t)$

same as

B(Drew) OR B(Peter) OR B(Keshav) OR…OR B(Michaela)

Existential Quantifier

Let x, y range over $\mathbb{N}$

$Q(y) ::= \exists x.\ x < y$

$Q(3)$ is T ([$x<3$] is T for $x=1$)
$Q(1)$ is T ([$x<1$] is T for $x=0$)
$Q(0)$ is F ([$x<0$] is not T for any x in $\mathbb{N}$)

Universal Quantifier

x, y range over $\mathbb{N}$

$R(y) ::= \forall x.\ x < y$

$R(1)$ is F ([$x<1$] is F for $x=5$)
$R(8)$ is F ([$x<8$] is F for $x=12$)
$R(10^{100})$ is F ([$x<10^{100}$] is F for $x=10^{100}$)

virus attack, I: $\forall\exists$

~~$\forall v \in$ virus . $\exists d \in$ defense.~~
d protects against v

For every virus, I have a defense:
against MYDOOM, use Defender
against ILOVEYOU, use Norton
against BABLAS, use Zonealarm…

$\forall\exists$ is expensive!

virus attack, II: $\exists\forall$

$\exists d \in$ defense . $\forall v \in$ virus.
d protects against v

That's what we want!

Example: d is MITviruscan, protects against *all* viruses

Alternating Quantifiers

$G ::= \forall x \exists y.\ x < y$

x, y range over Domain of Discourse

Domain	G is:
$\mathbb{N}$	T
ints < 0	F
reals < 0	T

Reverse the Quantifiers

$H ::= \exists y \forall x.\ x \leq y$

Domain	H is:
$\mathbb{N}$	F
$\mathbb{Z}^-$	T
$\mathbb{R}^-$	F