Mathematics for Computer Science

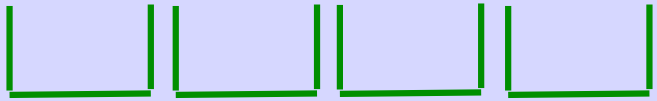
MIT 6.042J/18.062J

The Pigeonhole Principle

Pigeonhole Principle

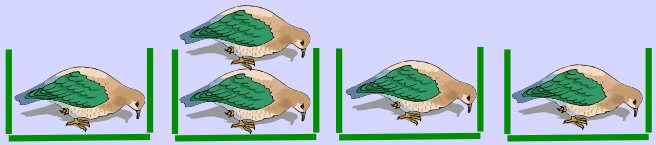
If more pigeons

than pigeonholes,



Pigeonhole Principle

then some hole must have $\geq$ two pigeons!



Pigeonhole Principle

Mapping Rule: total injection from A to B implies $|A| \leq |B|$.

If $|A| > |B|$, then
no total injection from A to B.

example: 5 Card Draw

set of 5 cards:

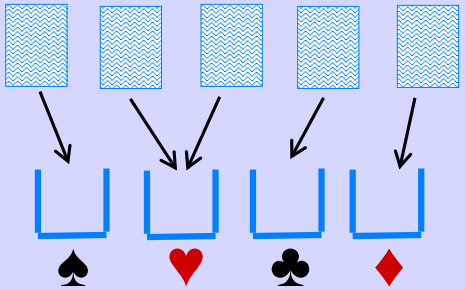
must have ≥ 2

with the same suit.

5 Card Draw

5 cards (pigeons)

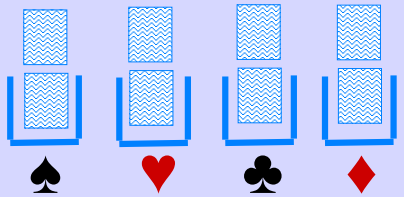
4 suits (holes)



10 Card Draw

10 cards: how many have the same suit?

10 Card Draw



< 3 cards in every hole?

10 Card Draw

cards with same suit

$$\geq \left\lceil \frac{10}{4} \right\rceil = 3$$

"ceiling," means round up

Generalized Pigeonhole Principle

If n pigeons and h holes, then some hole has $\geq \left\lceil \frac{n}{h} \right\rceil$ pigeons.