

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

*Mathematics for Computer Science*  
MIT 6.042J/18.062J

# Parallel Scheduling

Albert R Meyer March 20, 2013

scheduling.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

**parallel processing time**

min # terms to graduate:

**min parallel time = max chain size**

max term load:

**# processors for min time**

**$\leq$  max antichain size**

5 in this case

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paralleltime.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

**Minimum "Parallel" Time**

**#processors**

**$\leq$  max antichain size**

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paralleltime.3

6	9	13	7
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15	8	11	2

**Minimum "Parallel" Time**

**#processors**

**$\leq$  max antichain size**

we saw 3 processors

may still do min time

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paralleltime.4

6	9	13	7
12		10	5
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15	8	11	2


### Minimum # terms?

13 subjects

max chain size = 5

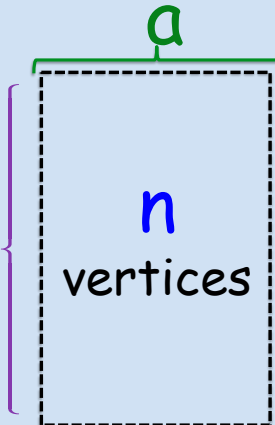
so load of some term must be

$$\geq \left\lceil \frac{13}{5} \right\rceil = 3$$

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### max antichain size



max chain size  $c$

$n \leq c \cdot a$

not both small

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### Dilworth's Lemma

every  $n$ -vertex DAG has

- a chain of size  $> t$
- or antichain of size  $\geq \frac{n}{t}$

for all  $1 \leq t \leq n$ .


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6	9	13	7
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### Dilworth's Lemma

every  $n$ -vertex DAG has

- a chain of size  $> \sqrt{n}$
- or antichain of size  $\geq \sqrt{n}$

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## Height/Birthday DAG

Edge from one student to another iff one is **shorter** and **younger** than the other.

$$(s_1, a_1) \rightarrow (s_2, a_2)$$

iff  $(s_1 \leq s_2)$  and  $(a_1 \leq a_2)$   
(the **product** graph)

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March 20, 2013
parallelttime.9

6	9	13	7
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## Height/Birthday DAG

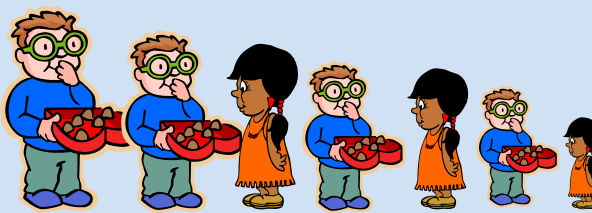
by Dilworth, our class of 141 has a chain or antichain of

$$\left\lceil \sqrt{141} \right\rceil = 12$$

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parallelttime.10

6	9	13	7
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## Dilworth Lemma



older

a height/bday antichain

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## height/bday antichain

Banda Daltso:	9/11/91, 5'	
Biswas Jyotizhka:	4/9/92, 5'1''	
Felso Valkyrie:	4/20/92, 5'1''	
Balewski Zuzanna:	8/5/92, 5'2''	
Chang Carolyn:	9/5/92, 5'2''	
Che Denise:	3/19/93, 5'3''	
Bartel Kathryn:	5/24/93, 5'4''	
Andersen Jessica:	8/6/93, 5'5''	
Abate Shalom:	9/3/93, 5'9''	
Batscha Jonathan:	11/8/93, 5'10''	
Alowayed Yousef:	4/4/94, 6'	
Belson Itamar:	8/30/94, 6'1''	



older  
shorter

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parallelttime.12