

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science  
6.042J/18.062J

# Propositional Normal Forms



Albert R Meyer

February 12, 2018

DNF-CNF.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Formulas for Truth Tables

There is a propositional formula for every truth table. In fact, there is a special **sum of products** formula.



Albert R Meyer

February 12, 2018

CNF-DNF.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Sum of Products Formula  
OR of ANDs

Disjunctive  
Normal Form  
- DNF



Albert R Meyer

February 12, 2018

DNF-CNF.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Majority function $M$

P	Q	R	$M(P,Q,R)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F



Albert R Meyer

February 12, 2018

DNF-CNF.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## AND-of-literals

Row **TTF** has value **T**

**P AND Q AND  $\bar{R}$**

is a term that is **True**  
in that row only



Albert R Meyer

February 12, 2018

DNF-CNF.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## AND-of-literals

**T** in the row **TTF** only

**P AND Q AND  $\bar{R}$**   
 $\underbrace{\hspace{10em}}$   
**A TTF**



Albert R Meyer

February 12, 2018

DNF-CNF.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## True rows of **M**

**TTT, TTF, TFT, FTT**

**A<sup>TTT</sup> OR A<sup>TTF</sup> OR**

**A<sup>TFT</sup> OR A<sup>FTT</sup>**

is **T** in exactly these rows



Albert R Meyer

February 12, 2018

DNF-CNF.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## DNF for **M**

**A<sup>TTT</sup> OR A<sup>TTF</sup> OR**

**A<sup>TFT</sup> OR A<sup>FTT</sup>**



Albert R Meyer

February 12, 2018

DNF-CNF.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Full DNF for M

$(P \text{ AND } Q \text{ AND } R)$  OR  
 $(P \text{ AND } Q \text{ AND } \bar{R})$  OR  
 $(P \text{ AND } \bar{Q} \text{ AND } R)$  OR  
 $(\bar{P} \text{ AND } Q \text{ AND } R)$

each product has all variables



Albert R Meyer

February 12, 2018

DNF-CNF.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Corollary

Every formula is equivalent to a Full DNF



Albert R Meyer

February 12, 2018

DNF-CNF.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Every formula  $\equiv$  Full DNF

Read the Full DNF right off the T rows of the truth table



Albert R Meyer

February 12, 2018

DNF-CNF.11

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Every formula  $\equiv$  Full DNF

Corollary: If two formulas are equivalent then they have the same\* Full DNF

\*sorted



Albert R Meyer

February 12, 2018

DNF-CNF.12

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Product of Sums Form

AND of ORs

Conjunctive Normal Form (CNF)



Albert R Meyer

February 12, 2018

DNF-CNF.13

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Majority function $M$

P	Q	R	$M(P,Q,R)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F



Albert R Meyer

February 12, 2018

DNF-CNF.14

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## OR-of-literals

Row TFF has value F

$\bar{P} \text{ OR } Q \text{ OR } R$

is a term that is False  
in the TFF row only



Albert R Meyer

February 12, 2018

DNF-CNF.15

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## OR-of-literals

F in row TFF only

$\bar{P} \text{ OR } Q \text{ OR } R$   
D TFF



Albert R Meyer

February 12, 2018

DNF-CNF.16

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Product of Sums for  $M$   
using rows with value  $F$

$$D^{TFF} \text{ AND } D^{FTF} \text{ AND } D^{FFT} \text{ AND } D^{FFF}$$



Albert R Meyer

February 12, 2018

DNF-CNF.17

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

CNF for  $M$

$$D^{TFF} \text{ AND } D^{FTF} \text{ AND } D^{FFT} \text{ AND } D^{FFF}$$



Albert R Meyer

February 12, 2018

DNF-CNF.18

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Full CNF for  $M$

$$(\bar{P} \text{ OR } Q \text{ OR } R) \text{ AND } (P \text{ OR } \bar{Q} \text{ OR } R) \text{ AND } (P \text{ OR } Q \text{ OR } \bar{R}) \text{ AND } (P \text{ OR } Q \text{ OR } R)$$

each sum has all variables



Albert R Meyer

February 12, 2018

DNF-CNF.19

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Every formula is  
equivalent to a  
Full CNF



Albert R Meyer

February 12, 2018

DNF-CNF.20

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Every formula  $\equiv$  Full CNF

**Corollary:** If two formulas are equivalent then they have the same\* Full CNF

\*sorted



Albert R Meyer

February 12, 2018

DNF-CNF.21