Conflict Midterm Exam September 18

Your name:_____

Identify your Team: **1PM** **Table (A–J):** _____
2:30PM **Table (A–J, 1–13):** _____

- This exam is **closed book** except for a 2-sided cribsheet. Total time is 90 minutes.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem.
- In answering the following questions, you may use without proof any of the results from class or text.

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	15		
2	25		
3	20		
4	20		
5	20		
Total	100		

Problem 1 (Irrational log) (15 points).
Prove that $\log_{12} 18$ is irrational.

Problem 2 (WOP) (25 points).
Prove by the Well Ordering Principle that for all nonnegative integers n

$$\sum_{i=0}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

Problem 3 (Satisfiable Propositions) (20 points).
For the following propositions, indicate whether they are valid (**V**), satisfiable but not valid (**S**), or not satisfiable (**N**). For propositions marked **S**, give an assignment of truth values that makes the proposition **True** and an assignment that makes it **False**.

(a) P IMPLIES $\overline{P}$

(b) P AND Q AND Z AND (P IMPLIES $\overline{Q}$)

(c) (P AND Q AND ($\overline{P}$ OR $\overline{Q}$)) IMPLIES [((Z AND (P IMPLIES Q)) OR (P AND $\overline{P}$))]

(d) (P AND Q) OR (P IMPLIES Q)

(e) (P OR $\overline{P}$ OR Q) IMPLIES (Q AND (Q IMPLIES $\overline{Q}$))

Problem 4 (Validity by Cases) (20 points).
There are many ways to show that the following formula is valid.

$$[(\overline{A} \text{ OR } \overline{B} \text{ OR } \overline{C} \text{ OR } \overline{E} \text{ OR } \overline{F}) \text{ AND } \overline{F}] \text{ OR } G \text{ OR } (\overline{H} \text{ AND } \overline{I} \text{ AND } \overline{J}) \text{ OR } K$$
$$\text{IMPLIES}$$
$$\text{NOT}(\overline{X} \text{ AND } \overline{Y} \text{ AND NOT}(X \text{ IMPLIES } Z))$$

(a) Laborious Lucy decides to verify this formula by truth table. Write a simple numerical expression for the number of rows she will have in her truth table. (You need not evaluate your expression.)

(b) You decide to be a little more efficient. Verify that the formula is valid by reasoning by cases according to the truth value of X.

Problem 5 (Normal Forms) (20 points).

(a) Bernardo, Francisco, Marcellus and Horatio are standing guard on the tower wall when they hear a ghost. Someone needs to leave to investigate the ghost, but someone needs to keep standing guard on the tower wall. Let B be the proposition that **B**ernardo will leave to investigate the ghost, F that **F**rancisco will leave to investigate, M that **M**arcellus will leave to investigate, and H that **H**oratio will leave to investigate.

Using these propositions, translate the following sentence directly into a Full[1] Conjunctive Normal Form propositional formula

At least one guard will leave to investigate, and at least one guard will not leave. (*)

(b) How many AND-of-literals subformulas are there in the Full[2] **Dis**junctive Normal Form that expresses (*)?

[1] "Full" means each OR term contain all of B, F, M, and H.

[2] "Full" means each AND term contain all of B, F, M, and H.