Conflict Midterm Exam September 18

Your name: ____________________________

Identify your Team: 1PM Table (A–J): ______ 2:30PM Table (A–J, 1–13): ______

• This exam is closed book except for a 2-sided cribsheet. Total time is 90 minutes.

• Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem.

• In answering the following questions, you may use without proof any of the results from class or text.

DO NOT WRITE BELOW THIS LINE

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Problem 1 (Irrational log) (15 points).
Prove that $\log_{12} 18$ is irrational.
Problem 2 (WOP) (25 points).
Prove by the Well Ordering Principle that for all nonnegative integers \( n \)
\[
\sum_{i=0}^{n} i^3 = \left( \frac{n(n + 1)}{2} \right)^2.
\]
Problem 3 (Satisfiable Propositions) (20 points).
For the following propositions, indicate whether they are valid (V), satisfiable but not valid (S), or not satisfiable (N). For propositions marked S, give an assignment of truth values that makes the proposition True and an assignment that makes it False.

(a) $P \implies \overline{P}$

(b) $P \land Q \land Z \land (P \implies \overline{Q})$

(c) $(P \land Q \land (\overline{P} \lor \overline{Q})) \implies [(Z \land (P \implies Q)) \lor (P \land \overline{P})]$ 

(d) $(P \land Q) \lor (P \implies Q)$

(e) $(P \lor \overline{P} \lor Q) \implies (Q \land (Q \implies \overline{Q}))$
Problem 4 (Validity by Cases) (20 points).
There are many ways to show that the following formula is valid.

\[
[(\overline{A} \lor \overline{B} \lor \overline{C} \lor \overline{E} \lor \overline{F}) \land \overline{F}] \lor \overline{G} \lor (\overline{H} \land \overline{I} \land \overline{J}) \lor K
\]

\[
\text{implies}
\]

\[
\overline{X} \land \overline{Y} \land \overline{X \ implies \ Z}
\]

(a) Laborious Lucy decides to verify this formula by truth table. Write a simple numerical expression for the number of rows she will have in her truth table. (You need not evaluate your expression.)

(b) You decide to be a little more efficient. Verify that the formula is valid by reasoning by cases according to the truth value of \(X\).
Problem 5 (Normal Forms) (20 points).

(a) Bernardo, Francisco, Marcellus and Horatio are standing guard on the tower wall when they hear a ghost. Someone needs to leave to investigate the ghost, but someone needs to keep standing guard on the tower wall. Let $B$ be the proposition that Bernardo will leave to investigate the ghost, $F$ that Francisco will leave to investigate, $M$ that Marcellus will leave to investigate, and $H$ that Horatio will leave to investigate. Using these propositions, translate the following sentence directly into a Full Conjunctive Normal Form propositional formula

At least one guard will leave to investigate, and at least one guard will not leave. (*)

(b) How many AND-of-literals subformulas are there in the Full Disjunctive Normal Form that expresses (*)?

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1.“Full” means each OR term contain all of $B, F, M,$ and $H.$
2.“Full” means each AND term contain all of $B, F, M,$ and $H.$