Mathematics for Computer Science

MIT 6.042J/18.062J

Magic Card Trick, II

A Magic Trick

audience chooses 5 cards

Assistant reveals 4 of them

Magician announces 5th card!

Magic Trick Revealed (I)

Among 5 cards chosen:

at least 2 have the same suit
(*Pigeonhole Principle*)

A hides one, lists the other one 1st

Aha! The first card has the same suit as the hidden card!

Magic Trick Revealed (II)

How does M figure out the rank of the hidden card?

Aha! Look at the order of the other 3 cards!

Magic Trick Revealed (II)

Fix ordering of the deck

A♣ < A♦ < A♥ < A♠ <

2♣ < 2♦ < 2♥ < 2♠ <

⋮

K♣ < K♦ < K♥ < K♠

Magic Trick Revealed (II)

Possible orders for the remaining 3 cards:

{ **SML**, **SLM**, **MSL**, **MLS**, **LSM**, **LMS** }

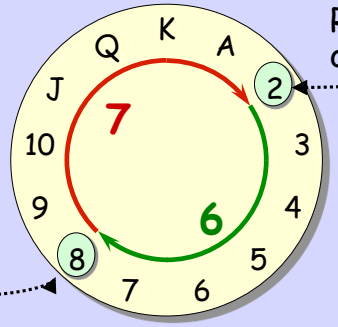
Magic Trick Revealed (II)

Wait! **Only have 6 sequences of the remaining 3 cards, but 12 possible hidden cards of the known suit!**

*Of **two** cards with the same suit, choosing **which** to reveal can give 1 more bit of information!*
Aha!

Clockwise Distance

The *smaller* clockwise distance between 2 card ranks is at most **6**:



Reveal the other card

Hide card with smaller offset.

Magic Trick Revealed (Finally)

- The first card determines the hidden suit (♠ ♥ ♦ ♣) .
- Hidden rank (A … K) = first-card rank + offset (≤ 6).
- Offset given by order of remaining 3 cards:

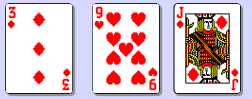
SML = 1, **SLM** = 2, **MSL** = 3,
MLS = 4, **LSM** = 5, **LMS** = 6.

Example

same suit

First: 

Hidden: 

Offset = **1** = **SML**: 

won't work with 4-card hands

audience can pick

$\binom{52}{4} = 270{,}725$

possible 4-card hands

A can reveal

$\frac{52!}{49!} = 132{,}600$

possible 3-card lists

won't work with 4 card hands

so at least

$$\left\lceil \frac{270{,}725}{132{,}600} \right\rceil = 3$$

hands map to the same list
– M can't tell which!