

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science  
MIT 6.042J/18.062J

# Magic Card Trick, I



Albert R Meyer, April 24, 2013

magictrickI.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## A Magic Trick

audience chooses 5 cards  
Assistant reveals 4 of them  
Magician announces 5<sup>th</sup> card!



Albert R Meyer, April 24, 2013

magictrickI.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Example

audience picks:



assistant shows:



magician announces:



Albert R Meyer, April 24, 2013

magictrickI.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Assistant's Choices

Decide the order of the 4 cards:  
 $4! = 24$  orderings  
-- but 48 cards remain  
Decide which 4 cards to list













Albert R Meyer, April 24, 2013

magictrickI.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2


## Map hands to 4-Card lists

*5-card hands (no order)*  ?  *4-card lists (ordered)*

list must come from hand



Which one to pick?






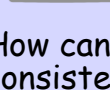
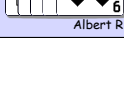



Albert R Meyer, April 24, 2013 magictrickI.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Map hands to 4-Card lists

*5-card hands (no order)*  ?  *4-card lists (ordered)*

How can we ensure consistency?

Albert R Meyer, April 24, 2013 magictrickI.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Map hands to 4-Card lists

*5-card hands (no order)*  *4-card lists (ordered)*

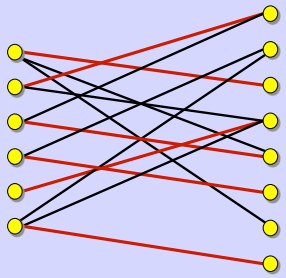
Every hand must have an identifying list!

Albert R Meyer, April 24, 2013 magictrickI.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## perfect matching of the hands

...is what we need

*5-card hands (no order)*  *4-card lists (ordered)*

Every hand must have an identifying list!

Albert R Meyer, April 24, 2013 magictrickI.9

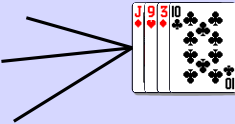
6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Match hands with 4-Card lists



$$\text{deg} = 5 \cdot 4 \cdot 3 \cdot 2 = 120$$

$$\text{deg} = 52 - 4 = 48$$



Albert R Meyer, April 24, 2013

magictrickI.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Match hands with 4-Card lists

So graph is  
**degree-constrained**  
 and hence has a matching  
 that A & M can use



Albert R Meyer, April 24, 2013

magictrickI.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## A Memorable Matching?

$$\binom{52}{5} = 2,598,960 \text{ hands to match}$$

How will A & M  
 learn any matching this big?

**next: How to do it**



Albert R Meyer, April 24, 2013

magictrickI.12