Mathematics for Computer Science

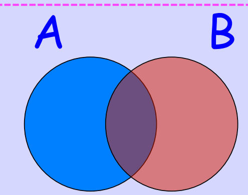
MIT 6.042J/18.062J

Inclusion-Exclusion

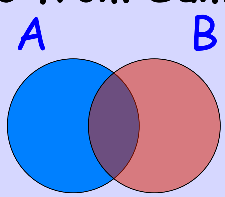
2 set proof

Inclusion-Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|$$



Inc-Exc from Sum Rule

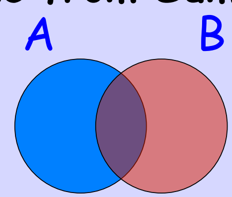


proof:

$$A \cup B = A \cup (B - A)$$

disjoint

Inc-Exc from Sum Rule

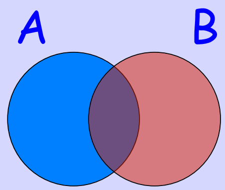


proof:

$$|A \cup B| = |A| + |B - A|$$

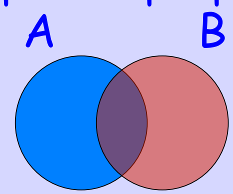
by Sum Rule

Inc-Exc from Sum Rule



$$|A \cup B| = |A| + \underbrace{|B - A|}_{|B| - |A \cap B|}$$

Lemma: $|B - A| = |B| - |A \cap B|$

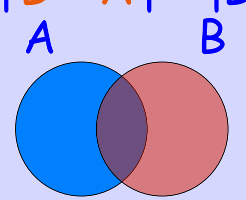


proof:

$$B = (B \cap A) \cup (B - A)$$

disjoint

Lemma: $|B - A| = |B| - |A \cap B|$



QED

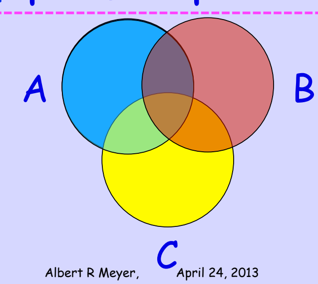
proof:

$$|B| = |B \cap A| + |B - A|$$

by Sum Rule

Inclusion-Exclusion (3 Sets)

$$\begin{aligned} |A \cup B \cup C| = \; & |A| + |B| + |C| \\ & - |A \cap B| - |A \cap C| - |B \cap C| \\ & + |A \cap B \cap C| \end{aligned}$$



Incl-Excl (n sets)

$$|A_1 \cup A_2 \cup \cdots \cup A_n| = \sum_{\emptyset \neq S \subseteq \{1,2,\ldots,n\}} (-1)^{|S|+1} \left| \bigcap_{i \in S} A_i \right|$$

Incl-Excl Formula: Proofs

by induction on n

--uninformative

by binomial counting

--next

by distributivity

--also