

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Inclusion-Exclusion 2 set proof



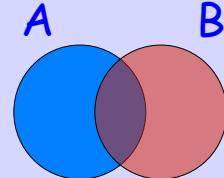
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incexcI.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Inclusion-Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|$$

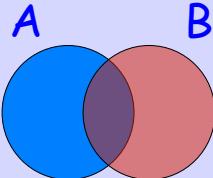


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incexcI.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Inc-Exc from Sum Rule



proof:

$$A \cup B = A \cup (B - A)$$



disjoint

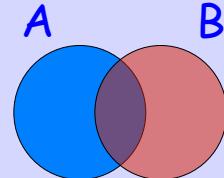


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incexcI.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Inc-Exc from Sum Rule



proof:

$$|A \cup B| = |A| + |B - A|$$

by Sum Rule

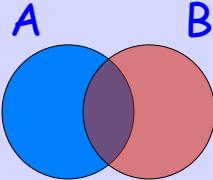


Albert R Meyer, April 24, 2013

incexcI.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Inc-Exc from Sum Rule



$$|A \cup B| = |A| + |B - A| \underbrace{|B| - |A \cap B|}_{|B| - |A \cap B|}$$

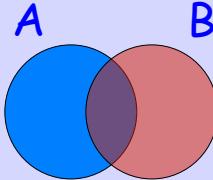


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incexcI.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Lemma: $|B - A| = |B| - |A \cap B|$



proof:

$$B = (B \cap A) \cup (B - A)$$

disjoint

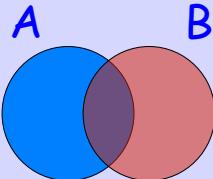


Albert R Meyer, April 24, 2013

incexcI.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Lemma: $|B - A| = |B| - |A \cap B|$



QED

proof:

$$|B| = |B \cap A| + |B - A|$$

by Sum Rule



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incexcI.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

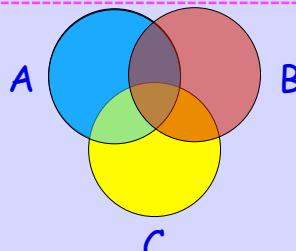
Inclusion-Exclusion (3 Sets)

$$\begin{aligned} |A \cup B \cup C| &= \\ &|A| + |B| + |C| \\ &- |A \cap B| - |A \cap C| - |B \cap C| \\ &+ |A \cap B \cap C| \end{aligned}$$



Albert R Meyer, April 24, 2013

incexcI.8



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Incl-Excl (n sets)

$$|A_1 \cup A_2 \cup \dots \cup A_n| = \sum_{\emptyset \neq S \subseteq \{1, 2, \dots, n\}} (-1)^{|S|+1} \left| \bigcap_{i \in S} A_i \right|$$



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incexcI.10

6	9	13	7
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15	8	11	2

Incl-Excl Formula: Proofs

by induction on n

--uninformative

by binomial counting

--next

by distributivity

--also



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incexcI.13