

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Inclusion-Exclusion 6042



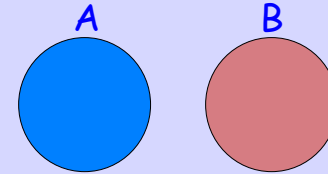
Albert R Meyer, April 24, 2013

incexc6042.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Sum Rule

$$|A \cup B| = |A| + |B|$$



for disjoint sets A, B



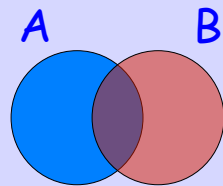
Albert R Meyer, April 24, 2013

incexc6042.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Sum Rule

$$|A \cup B| = ?$$



What if **not** disjoint?



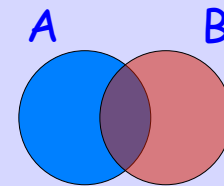
Albert R Meyer, April 24, 2013

incexc6042.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Inclusion-Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|$$



Albert R Meyer, April 24, 2013

incexc6042.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

permutations of [0,9]

0123456789

0134256789



Albert R Meyer, April 24, 2013

incexc6042.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

| permutations of [0,9] | = 10!

How many have 60, 04, or 42?

0123456789

0134256789



Albert R Meyer, April 24, 2013

incexc6042.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

| permutations of [0,9] | = 10!

How many have 60, 04, or 42?

0123456789 no

0134256789 yes

0624136789 no

1304256789 yes



Albert R Meyer, April 24, 2013

incexc6042.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

| permutations of [0,9] | = 10!

$P_x ::=$ perms with subsequence x

0123456789 no

0134256789 yes

0624136789 no

1304256789 yes



Albert R Meyer, April 24, 2013

incexc6042.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

| permutations of [0,9] | = 10!
 $P_x ::=$ perms with subsequence x

0123456789 no
 0134256789 $\in P_{42}$
 0624136789 no
 1304256789 $\in P_{04} \cap P_{42}$



Albert R Meyer,

April 24, 2013

incexc6042.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

$$|P_{60} \cup P_{04} \cup P_{42}| = ?$$



Albert R Meyer,

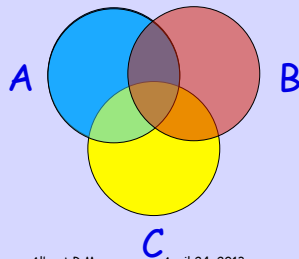
April 24, 2013

incexc6042.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Inclusion-Exclusion (3 Sets)

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$



Albert R Meyer,

April 24, 2013

incexc6042.11

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

$$|P_{60} \cup P_{04} \cup P_{42}| = |P_{60}| + |P_{04}| + |P_{42}| - |P_{60} \cap P_{04}| - |P_{60} \cap P_{42}| - |P_{04} \cap P_{42}| + |P_{60} \cap P_{04} \cap P_{42}|$$



Albert R Meyer,

April 24, 2013

incexc6042.12

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

$$|P_{60}| = |\text{perms}\{1,2,3,4,5,\underline{60},7,8,9\}|$$

$$= 9!$$

$$|P_{60} \cap P_{42}| =$$

$$|\text{perms}\{1,3,\underline{42},5,\underline{60},7,8,9\}| = 8!$$

$$|P_{60} \cap P_{04}| = |P_{604}| =$$

$$|\text{perms}\{1,2,3,5,\underline{604},7,8,9\}| = 8!$$



Albert R Meyer, April 24, 2013

incexc6042.13

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

$$|P_{60} \cap P_{04} \cap P_{42}| = |P_{6042}| =$$

$$|\text{perms}\{1,3,5,\underline{6042},7,8,9\}|$$

$$= 7!$$



Albert R Meyer, April 24, 2013

incexc6042.14

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

$$|P_{60} \cup P_{04} \cup P_{42}| =$$

$$|P_{60}| + |P_{04}| + |P_{42}|$$

$$- |P_{60} \cap P_{04}| - |P_{60} \cap P_{42}| - |P_{04} \cap P_{42}|$$

$$+ |P_{60} \cap P_{04} \cap P_{42}|$$



Albert R Meyer, April 24, 2013

incexc6042.15

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

$$|P_{60} \cup P_{04} \cup P_{42}| =$$

$$3 \cdot 9!$$

$$- |P_{60} \cap P_{04}| - |P_{60} \cap P_{42}| - |P_{04} \cap P_{42}|$$

$$+ |P_{60} \cap P_{04} \cap P_{42}|$$



Albert R Meyer, April 24, 2013

incexc6042.16

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

$$|P_{60} \cup P_{04} \cup P_{42}| =$$

$$3 \cdot 9!$$

$$- 3 \cdot 8!$$

$$+ |P_{60} \cap P_{04} \cap P_{42}|$$



Albert R Meyer, April 24, 2013

incexc6042.17

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

$$|P_{60} \cup P_{04} \cup P_{42}| =$$

$$3 \cdot 9!$$

$$- 3 \cdot 8!$$

$$+ 7!$$



Albert R Meyer, April 24, 2013

incexc6042.18

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: digit permutations

$$|P_{60} \cup P_{04} \cup P_{42}| =$$

$$193 \cdot 7! = 972720$$

$\approx 27\%$ of all perms of $[0,9]$



Albert R Meyer, April 24, 2013

incexc6042.19