Sums & Money

Geometric Series

\[ G_n = 1 + x + x^2 + \cdots + x^n \]
\[ -xG_n = -x - x^2 - \cdots - x^n - x^{n+1} \]
Geometric Series

\[ G_n = 1 + x + x^2 + \ldots + x^n \]

\[ -xG_n = -x - x^2 - \ldots - x^n - x^{n+1} \]

\[ G_n - xG_n = \frac{1}{1 - x} - x^{n+1} \]

Consider infinite sum (series)

\[ 1 + x + x^2 + \ldots + x^{n-1} + x^n + \ldots = \sum_{i=0}^{\infty} x^i \]
Infinite Geometric Series

\[ G_n = \frac{1-x^{n+1}}{1-x} \]

\[ \lim_{n \to \infty} G_n = \frac{1-\lim_{n \to \infty} x^{n+1}}{1-x} = \frac{1}{1-x} \]

for \(|x| < 1\)

The future value of $$

I will pay you $100 in 1 year, if you will pay me $X now.

My bank will pay me 3% interest. define \textit{bankrate}

\[ b ::= 1.03 \]

bank increases my $$ by this factor in 1 year.
The future value of $$
If I deposit your $$X$$ now, I will have $$b \cdot X$$ in 1 year.
So I won’t lose money as long as 
$$b \cdot X \geq 100$$
$$X \geq \frac{100}{1.03} \approx \$97.09$$

The future value of $$
- $1 in 1 year is worth $0.9709 now.
- $r$ last year is worth $1$ today, where $$r := \frac{1}{b}$$.
- So $$n$$ paid in 2 years is worth $$nr$$ paid in 1 year, and is worth $$nr^2$$ today.

The future value of $$
$n$ paid $k$ years from now is worth $$n \cdot r^k$$ today
where $$r := \frac{1}{\text{bankrate}}$$.

Annuities
I pay you $100/year for 10 years, if you will pay me $$Y$$ now.
I can’t lose if you pay me
$$100r + 100r^2 + 100r^3 + \cdots + 100r^{10}$$
$$= 100r(1 + r + \cdots + r^9)$$
$$= 100r(1 - r^{10})/(1 - r) = \$853.02$$
Annuities

I pay you $100/year for 10 years, if you will pay me $853.02.

QUICKIE: If bank rates unexpectedly increase in the next few years,

A. You come out ahead
B. The deal stays fair
C. I come out ahead

Manipulating Sums

\[ \sum_{i=0}^{n+1} x^i = \frac{1-x^{n+1}}{1-x} \]

\[ \sum_{i=0}^{n} i x^{i-1} = \frac{x}{x} \sum_{i=1}^{n} i x^i = \frac{d}{dx} \left( \frac{1-x^{n+1}}{1-x} \right) \]