Generalized Counting Rules

# lineups of 5 students in class
let $S := \text{students}$
say $|S| = 91$ so
$|\text{lineups of 5 students}| \neq 91$
student can’t be in 2 places:
$|\text{seqs in } S^5 \text{ with no repeats}| \neq 91 

Generalized Product Rule

Let $Q$ be a set of length-$k$ sequences
if $n_1$ possible 1\textsuperscript{st} elements,
$n_2$ possible 2\textsuperscript{nd} elements
(for each first entry),
n3 possible 3\textsuperscript{rd} elements
(for each 1\textsuperscript{st} & 2\textsuperscript{nd} entry, ...)
then, $|Q| = n_1 \cdot n_2 \cdots n_k$
Division Rule

#6.042 students = 
#6.042 students' fingers

\[ \frac{10}{10} \]

if function from A to B is \textit{k-to-1}, then

\[ |A| = k|B| \]

(generalized Bijection Rule)

Division Rule

function from A to B

\[ \text{# arrows} = |A| \]

Division Rule

function is \textit{k-to-1}

\[ \text{# arrows} = k|B| \]
Counting Subsets

How many size 4 subsets of \(\{1,2,\ldots,13\}\)?

\(\{1,2,3,4\}\)
\(\{1,2,3,5\}\)
\(\{3,4,7,11\}\)

\(\vdots\)

Counting Subsets

Let 
\(A::=\) permutations of \(\{1,2,\ldots,13\}\)

\(B::=\) size 4 subsets

map \(a_1 a_2 a_3 a_4 a_5\ldots a_{12} a_{13} \in A\)
to \(\{a_1, a_2, a_3, a_4\} \in B\)

\(\begin{align*}
\text{a}_1 \text{a}_3 \text{a}_2 \text{a}_4 \ldots \text{a}_{12} \text{a}_{13} \text{ also maps to } \{\text{a}_1, \text{a}_2, \text{a}_3, \text{a}_4\}
\text{so does } \text{a}_1 \text{a}_3 \text{a}_2 \text{a}_4 \text{a}_{13} \ldots \text{a}_{12} \text{a}_5
\text{4! perms 9! perms}
\text{all map to same set}
\end{align*}\)

\(4! \cdot 9!\)-to-1
Counting Subsets

The number of $m$ element subsets of an $n$ element set is given by the binomial coefficient:

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}$$

This is also known as "n choose m."