Proof by **Cases**:
**Friends & Strangers**

**Claim:** there is a set of
- 3 mutual friends or
- 3 mutual strangers

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**Friends & Strangers**

People are circles
- 3 mutual strangers
- 3 mutual friends

**red line shows friends**
**blue line shows strangers**

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**Friends & Strangers**

Take 3 minutes to find a counter-example
--or convince yourself there isn't any counterexample, that is, the **Claim** is true.
A Proof of the Claim

- Person ● has a line to each of the 5 other people.
- lines are red or blue, so at least 3 must be the same color.

\[
\text{● has } \geq 3 \text{ friends}
\]

Case 1: some pair of these friends are friends of each other, then we have 3 mutual friends:

Case 2: no pair of these friends are friends of each other, so we have 3 mutual strangers:

Since the Claim is true in either case, and one of these cases always holds, the Claim is always true.

\[\text{QED}\]
Ramsey's Theorem
For any $k$, every large enough group of people will include either $k$ mutual friends, or $k$ mutual strangers.

Ramsey's Theorem
For any $k$, every large enough group of people will include either size-$k$ red clique, or size-$k$ blue clique.

Let $R(k)$ be the large enough size.
So we've proved that $R(3) \leq 6$.

Ramsey's Numbers
Turns out that $R(4) = 18$ (not easy!)
$R(5)$ is unknown!
Paul Erdős considered finding $R(6)$ a hopeless challenge!
So in our second class, we have reached a research frontier!