Final Examination, Unit 3

Your name: ____________________

- This exam is **closed book** except for two 2-sided cribsheets. Total time is 180 minutes.
- Write your solutions in the space provided. If you need more space, **write on the back** of the sheet containing the problem.
- In answering the following questions, you may use without proof any of the results from class or text.

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	15		
2	20		
3	15		
Total	50		

Problem 1 (**Cardinality**) (**15 points**). **(a)** For each of the following sets, indicate whether it is finite (**F**), countably infinite (**C**), or uncountable (**U**).

(i) The set of even integers greater than 10^{100}. ______

(ii) The set of "pure" complex numbers of the form ri for nonzero real numbers r. ______

(iii) The powerset of the integer interval $[10..10^{10}]$. ______

(iv) The complex numbers c such that c is the root of a quadratic with integer coefficients, that is, ______

$$\exists m, n, p \in \mathbb{Z}, m \neq 0.\ mc^2 + nc + p = 0.$$

Let $\mathcal{U}$ be an uncountable set, $\mathcal{C}$ be a countably infinite subset of $\mathcal{U}$, and $\mathcal{D}$ be a countably infinite set.

(v) $\mathcal{U} \cup \mathcal{D}$. ______

(vi) $\mathcal{U} \cap \mathcal{C}$ ______

(vii) $\mathcal{U} - \mathcal{D}$ ______

(b) Give an example of sets A and B such that

$$\mathbb{R} \text{ strict } A \text{ strict } B.$$

Problem 2 (**Coloring**) (**20 points**). **(a)** Determine a valid coloring of the graph G shown in Figure 1 using as few colors as possible. (Simply write your proposed color for each vertex next to that vertex. You may use R for red, G for green, etc.)

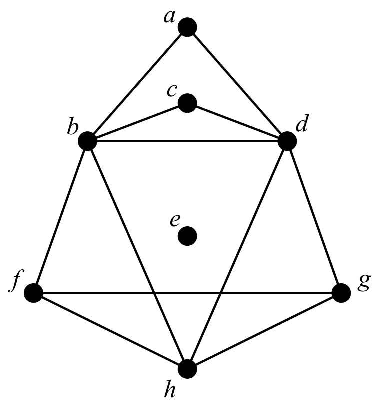

Figure 1 The simple graph G

(b) What is the chromatic number of the graph? Explain why your answer is correct.

(c) Is it possible to increase the chromatic number of the graph by adding just one edge? If yes, state which new edge would do the trick, and explain why. If no, explain why.

(d) Is it possible to decrease the chromatic number of the graph by removing just one edge? If yes, state which edge could be removed to do the trick, and explain why. If no, explain why.

Problem 3 (**Partial Orders**) (**15 points**).
In each of the following examples, R is a binary relation on a set A. Indicate which examples are weak partial orders or strong partial orders, and when they are, indicate whether they are linear. **Explain** your answers.

(a) $A = \{a, b, c\},\ R = \{(a, a), (b, a), (b, b), (b, c), (c, c)\}$.

(b) $A =$ the set of all English words,

$$R = \{(x, y) \in A \times A \mid x \text{ comes before } y \text{ alphabetically}\}.$$

(c) $A = \mathbb{R},\ R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid |x| < |y| \text{ OR } x = y\}$.

(d) $A = \mathbb{R},\ R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid |x| \leq |y|\}$.