

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Directed Graphs (Digraphs)

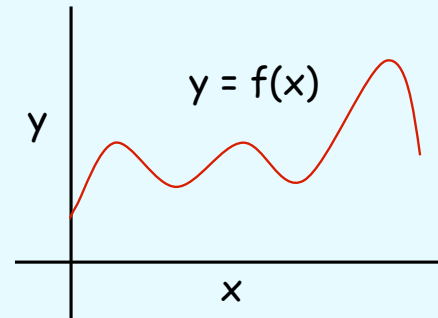


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digraphs.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Normal Person's Graph

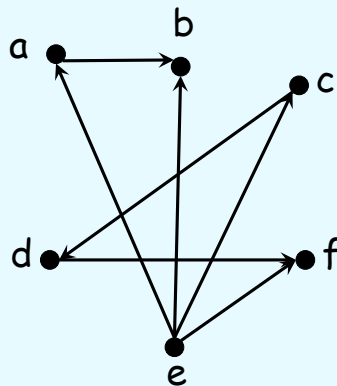


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digraphs.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Computer Scientist's Graph



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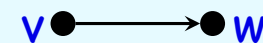
digraphs.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Digraphs

- a set, V , of vertices
- a set, $E \subseteq V \times V$ of directed edges

$(v, w) \in E$ notation: $v \rightarrow w$

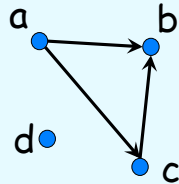


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digraphs.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Relations and Graphs



$$V = \{a, b, c, d\}$$

$$E = \{(a, b), (a, c), (c, b)\}$$



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digraphs.5

6	9	13	7
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Digraphs

Formally, a digraph with vertices V is *the same* as a binary relation on V .



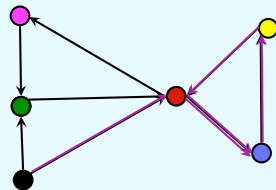
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digraphs.6

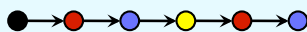
6	9	13	7
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Walks & Paths

Walk: follow successive edges



length: 5 edges



(not the 6 vertices)



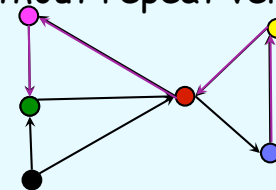
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digraphs.7

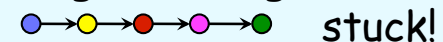
6	9	13	7
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Walks & Paths

Path: walk thru vertices without repeat vertex



length: 4 edges



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digraphs.8

6	9	13	7
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3	1	4	14
15	8	11	2

Matrix representation

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-

$$\left(\begin{array}{ccccccc} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{array} \right)$$

Albert R Meyer March 15, 2013 digraphs.9

6	9	13	7
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Matrix representation

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-
-
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$$\left(\begin{array}{ccccccc} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{array} \right)$$

Albert R Meyer March 15, 2013 digraphs.10

6	9	13	7
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Matrix representation

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-
-

$$\left(\begin{array}{ccccccc} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{array} \right)$$

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6	9	13	7
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Adjacency Matrix

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-

$$\left(\begin{array}{ccccccc} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{array} \right)$$

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6	9	13	7
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Adjacency Matrix

$$\begin{matrix} & \bullet & \color{red}\bullet & \color{blue}\bullet & \color{yellow}\bullet & \color{magenta}\bullet & \color{green}\bullet \\ \bullet & \left(\begin{array}{cccccc} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{array} \right) \end{matrix}$$

