

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

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# Number Theory: Die Hard



Albert R Meyer

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diehardprimes.1

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## Generalized Die Hard

Did it with buckets:

3 gal. & 5 gal.

3 gal. & 9 gal.

Now  $a$  gal. &  $b$  gal.?



Albert R Meyer

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diehardprimes.2

6	9	13	7
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## Generalized Die Hard

Under Die Hard rules,  
gal.'s in each bucket are

linear combinations

of  $a$  and  $b$



Albert R Meyer

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6	9	13	7
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## Generalized Die Hard

Under Die Hard rules,  
gal.'s in each bucket are

multiples of  $\gcd(a,b)$

of  $a$  and  $b$



Albert R Meyer

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diehardprimes.4

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## Generalized Die Hard

**Claim:** Can get **any linear combination** of **a, b** into a bucket (if there's room for it).  
 Namely, say  $0 \leq sa + tb < b$ .  
 Get  $sa + tb$  into the **b gal.** bucket as follows:



Albert R Meyer

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## Generalized Die Hard

assume  $s > 0$ . do  $s$  times:  
 fill bucket **a**, pour into **b**  
 — if **b** fills, empty it.  
 total fills =  $sa$   
 $0 \leq$  amount left  $< b$   
 # **b** emptyings must be  $-t$



Albert R Meyer

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## Generalized Die Hard

In fact, no need to count:  
 fill bucket **a**, pour into **b**  
 — if **b** fills, empty it  
 — until **desired gal.'s** in **b**!



Albert R Meyer

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diehardprimes.8