Mathematics for Computer Science

MIT 6.042J/18.062J
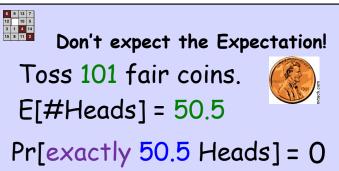
Deviation from the Mean

Don't expect the Expectation!

Toss 101 fair coins.

E[#Heads] = 50.5

Pr[exactly 50.5 Heads] = 0

Pr[exactly 50 Heads] < 1/13

Pr[50.5 $\pm$ 1 Heads]   < 1/7

Don't expect the Expectation!

Toss 1001 fair coins.

E[#Heads]   = 500.5

Pr[#H = 500]   < 1/39

Pr[#H = 500.5 $\pm$ 1] < 1/19

smaller

Don't expect the Expectation!

As #tosses grows, #Heads gets less likely to be within a *fixed* distance of the mean

1

Within a % of the mean?

Toss 1001 fair coins. of 1001

Pr[#H = 500.5 ± 1%]

= Pr[#H = 500.5 ± 10]

≈ 0.49

not so bad

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Giving Meaning to the Mean

Let $\mu ::= E[R]$. What is

Pr[R far from μ]?

Pr[$|R - \mu| > x$]

R's average deviation ?

E[$|R - \mu|$] ?

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Two Dice with Same Mean

Fair Die

• $E[D_1]$ = 3.5

Loaded Die throwing only 1 & 6:

• $E[D_2]$ = (1+6)/2 = 3.5 also!

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Two Dice with Same Mean

deviation from the mean

Fair

1.5 on average

Pr[D = i]

Loaded

2.5

i: 0 1 2 3 4 5 6 7

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Dice have Different Deviations

Fair Die:

$$E[\,|D_1 - \mu|\,] = 1.5$$

Loaded Die:

$$E[\,|D_2 - \mu|\,] = 2.5$$

Giving Meaning to the Mean

The mean alone is not a good predictor of R's behavior. We generally need more about its distribution, especially probable deviation from its mean.