Chebyshev Bound

\[ \Pr[|R - \mu| \geq x] \leq \frac{\text{Var}[R]}{x^2} \]

Chebyshev Bound

Pr[|R - \mu| \geq x] \leq \frac{\text{Var}[R]}{x^2}

Var[R] := E[(R - \mu)^2]

Variance of a Random Variable

Var[R] := E[(R - \mu)^2]

Variance is also called the mean square error

Improving the Markov Bound

\[ \Pr[|R - \mu| \geq x] = \Pr[(R - \mu)^2 \geq x^2] \]

by Markov:

\[ \frac{E[(R - \mu)^2]}{x^2} \leq \text{variance of } R \]
Chebyshev Bound

\[
\Pr[|R - \mu| \geq x] \leq \frac{\text{Var}[R]}{x^2}
\]

\[\sigma_R := \sqrt{\text{Var}[R]}\]
standard deviation

Standard Deviation of an RV

Standard deviation is also called the root mean square error

\[\sigma_R := \sqrt{\text{Var}[R]}\]
standard deviation

Standard Deviation of an RV

\[\sigma_R := \sqrt{\text{Var}[R]}\]

Chebyshev Bound

\[
\Pr[|R - \mu| \geq x] \leq \frac{\sigma_R^2}{x^2}
\]

\[\sigma_R := \sqrt{\text{Var}[R]}\]
Chebyshev Bound (Restated)

\[
\Pr[ |R - \mu| \geq c\sigma_R ] \leq \frac{1}{c^2}
\]

\[
\sigma_R \doteq \sqrt{\text{Var}[R]}
\]

Standard Deviation

\[
\Pr[ |R - \mu| \geq c\sigma_R ] \leq \frac{1}{c^2}
\]

\( R \) probably not many \( \sigma \)'s from \( \mu \):

- Further than \( \sigma \): \( \Pr \leq 1 \)
- Further than \( 2\sigma \): \( \Pr \leq 1/4 \)
- Further than \( 3\sigma \): \( \Pr \leq 1/9 \)
- Further than \( 4\sigma \): \( \Pr \leq 1/16 \)