Mathematics for Computer Science

MIT 6.042J/18.062J

Bounds on Deviation

Chebyshev Bound

Improving the Markov Bound

$$\Pr[|R-\mu| \geq x]$$
$$= \Pr[(R-\mu)^2 \geq x^2]$$

by Markov:

$$\leq \frac{E[(R-\mu)^2]}{x^2}$$

variance of R

Chebyshev Bound

$$\boxed{\Pr[\,|R-\mu| \geq x\,] \leq \frac{\mathrm{Var}[R]}{x^2}}$$

$$\mathrm{Var}[R] ::= E[(R-\mu)^2]$$

Variance of a Random Variable

$$\mathrm{Var}[R] ::= E[(R-\mu)^2]$$

Variance is also called the

mean square error

Chebyshev Bound

$$\Pr[\,|R-\mu| \geq x\,] \leq \frac{\mathrm{Var}[R]}{x^2}$$

$$\sigma_R ::= \sqrt{\mathrm{Var}[R]}$$

standard deviation

Standard Deviation of an RV

Standard deviation is also called the

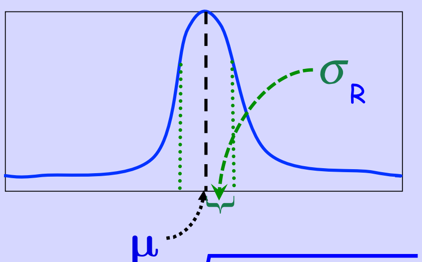
root mean square error

$$\sigma_R ::= \sqrt{\mathrm{Var}[R]}$$

standard deviation

Standard Deviation of an RV

PDF_R



$$\sigma_R ::= \sqrt{\mathrm{Var}[R]}$$

Chebyshev Bound

$$\Pr[\,|R-\mu| \geq x\,] \leq \frac{\sigma_R^2}{x^2}$$

$$\sigma_R ::= \sqrt{\mathrm{Var}[R]}$$

Chebyshev Bound (Restated)

$$\Pr[\,|R-\mu| \geq c\sigma_R\,] \leq \frac{1}{c^2}$$

$$\sigma_R ::= \sqrt{\mathrm{Var}[R]}$$

Standard Deviation

$$\Pr[\,|R-\mu| \geq c\sigma_R\,] \leq \frac{1}{c^2}$$

R probably not many σ's from μ:

further than	σ	$\Pr \leq 1$
	2σ	$\Pr \leq 1/4$
	3σ	$\Pr \leq 1/9$
	4σ	$\Pr \leq 1/16$