Mathematics for Computer Science
MIT 6.042J/18.062J

Derived Variables

Derived Variables

A derived variable, v, is a function assigning a "value" to each state:

v: States $\rightarrow$ Values

If Vals = $\mathbb{N}$, say v is "$\mathbb{N}$-valued" or "nonnegative-integer-valued"

Derived Variables

Robot on the grid example:

States = $\mathbb{N}^2$. Define the sum-value, σ, of a state:

$$\sigma(x,y) ::= x+y$$

an $\mathbb{N}$-valued derived variable

Derived Variables

Called derived to distinguish from actual variables that appear in a program.

For robot Actual: x, y
Derived: σ

Derived Variables

Another derived variable:

π ::= parity(σ)

π is {0,1}-valued

Derived Variables

For Fast Exp, have (actual) variable Z.

Proof of termination:

Z is strictly decreasing & $\mathbb{N}$-valued

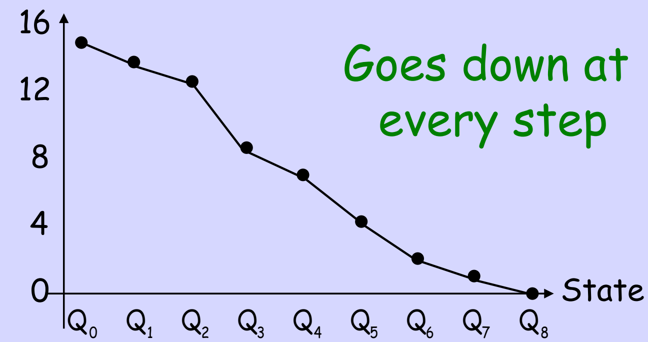
Derived Variables

Termination followed by Well Ordering Principle:

Z must take a least value.

then the algorithm is stuck

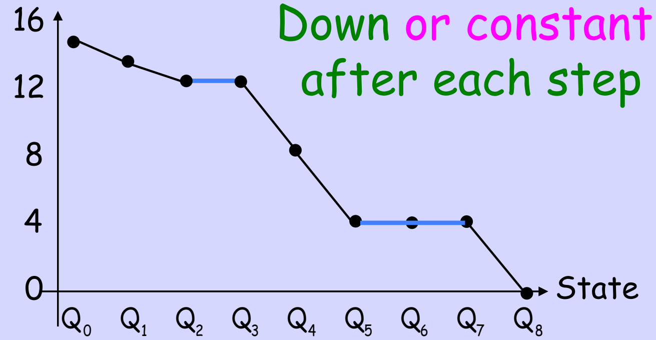
Strictly Decreasing Variable



Goes down at every step

Weakly Decreasing Variable

Down or constant after each step



Diagonal Robot variables

σ: up & down all over the place

neither increasing
nor decreasing

π: is constant

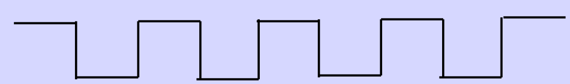
both weakly increasing
& weakly decreasing

Weakly Decreasing Variable

(We used to call weakly decreasing variables "nonincreasing" variables.)

Weakly Decreasing Variable

Caused confusion:
nonincreasing is
NOT SAME as "not increasing:"



Well ordered sets

Def. A set W of real numbers is well ordered

Well ordered sets

Def. A set W of real numbers is well ordered iff it has

NO infinite decreasing sequence

$$w_0 > w_1 > w_2 > \cdots > w_n > \cdots$$

Well ordered sets

Termination using WOP on $\mathbb{N}$ generalizes to strictly decreasing variables whose values are in any

well ordered set