A derived variable, \( v \), is a function assigning a “value” to each state:

\[ v: \text{States} \rightarrow \text{Values} \]

If \( \text{Vals} = \mathbb{N} \), say \( v \) is “\( \mathbb{N} \)-valued” or “nonnegative-integer-valued”

Robot on the grid example:
States = \( \mathbb{N}^2 \). Define the sum-value, \( \sigma \), of a state:

\[ \sigma(x,y) ::= x+y \]

an \( \mathbb{N} \)-valued derived variable

Called derived to distinguish from actual variables that appear in a program.
For robot Actual: \( x, y \)
Derived: \( \sigma \)
Another derived variable: 
\[ \pi := \text{parity}(\sigma) \]
\[ \pi \text{ is } \{0,1\}-\text{valued} \]

For Fast Exp, have (actual) variable Z. Proof of termination: 
Z is strictly decreasing & N-valued

Termination followed by 
Well Ordering Principle: 
Z must take a least value. 
then the algorithm is stuck

Goes down at every step
Weakly Decreasing Variable

Down or constant after each step

Diagonal Robot variables

\( \sigma \): up & down all over the place
neither increasing
nor decreasing

\( \pi \): is constant
both weakly increasing
& weakly decreasing

(We used to call weakly decreasing variables "nonincreasing" variables.)

Caused confusion:
nonincreasing is NOT SAME as "not increasing:"

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Albert R. Meyer          March 3, 2017
Well ordered sets

Def. A set $W$ of real numbers is well ordered iff it has NO infinite decreasing sequence $w_0 > w_1 > w_2 > \cdots > w_n > \cdots$.

Termination using WOP on $\mathbb{N}$ generalizes to strictly decreasing variables whose values are in any well ordered set.