In-Class Problems Week 5, Mon.

Problem 1.
Four Students want separate assignments to four VI-A Companies. Here are their preference rankings:

Student	Companies
Albert:	HP, Bellcore, AT&T, Draper
Sarah:	AT&T, Bellcore, Draper, HP
Tasha:	HP, Draper, AT&T, Bellcore
Elizabeth:	Draper, AT&T, Bellcore, HP

Company	Students
AT&T:	Elizabeth, Albert, Tasha, Sarah
Bellcore:	Tasha, Sarah, Albert, Elizabeth
HP:	Elizabeth, Tasha, Albert, Sarah
Draper:	Sarah, Elizabeth, Tasha, Albert

(a) Use the Mating Ritual to find *two* stable assignments of Students to Companies.

(b) Describe a simple procedure to determine whether any given stable marriage problem has a unique solution, that is, only one possible stable matching. Briefly explain why it works.

Problem 2.
The Mating Ritual for finding stable marriages works even when the numbers of men and women are not equal. As before, a set of (monogamous) marriages between men and women is called stable when it has no "rogue couples."

(a) Extend the definition of *rogue couple* so it covers the case of unmarried men and women. Verify that in a stable set of marriages, either all the men are married or all the women are married.

(b) Explain why even in the case of unequal numbers of men and women, applying the Mating Ritual will yield a stable matching.

Problem 3.
The most famous application of stable matching was in assigning graduating medical students to hospital residencies. Each hospital has a preference ranking of students, and each student has a preference ranking of hospitals, but unlike finding stable marriages between an equal number of boys and girls, hospitals generally have differing numbers of available residencies, and the total number of residencies may not equal the number of graduating students.

Explain how to adapt the Stable Matching problem with an equal number of boys and girls to this more general situation. In particular, modify the definition of stable matching so it applies in this situation, and explain how to adapt the Mating Ritual to handle it.

Problem 4.
The preferences among 4 boys and 4 girls are partially specified in the following table:

B1:	G1	G2	–	–
B2:	G2	G1	–	–
B3:	–	–	G4	G3
B4:	–	–	G3	G4
G1:	B2	B1	–	–
G2:	B1	B2	–	–
G3:	–	–	B3	B4
G4:	–	–	B4	B3

(a) Verify that

$$(B1, G1), (B2, G2), (B3, G3), (B4, G4)$$

will be a stable matching whatever the unspecified preferences may be.

(b) Explain why the stable matching above is neither boy-optimal nor boy-pessimal and so will not be an outcome of the Mating Ritual.

(c) Describe how to define a set of marriage preferences among n boys and n girls which have at least $2^{n/2}$ stable assignments.

Hint: Arrange the boys into a list of $n/2$ pairs, and likewise arrange the girls into a list of $n/2$ pairs of girls. Choose preferences so that the kth pair of boys ranks the kth pair of girls just below the previous pairs of girls, and likewise for the kth pair of girls. Within the kth pairs, make sure each boy's first choice girl in the pair prefers the other boy in the pair.