In-Class Problems Week 14, Wed.

Problem 1.
Suppose you flip three fair, mutually independent coins. Define the following events:

- Let A be the event that *the first* coin is heads.
- Let B be the event that *the second* coin is heads.
- Let C be the event that *the third* coin is heads.
- Let D be the event that *an even number of* coins are heads.

(a) Use the four step method to determine the probability space for this experiment and the probability of each of A, B, C, D.

(b) Show that these events are not mutually independent.

(c) Show that they are 3-way independent.

Problem 2.
A somewhat reliable allergy test has the following properties:

- If you are allergic, there is a 10% chance that the test will say you are not.
- If you are not allergic, there is a 5% chance that the test will say you are.

(a) The test results are correct at what confidence level?

(b) What is the Bayes factor for being allergic when the test diagnoses a person as allergic?

(c) What can you conclude about the odds of a random person being allergic given that the test diagnoses them as allergic? Can you determine if the odds are better than even?

Suppose that your doctor tells you that because the test diagnosed you as allergic, and about 25% of people are allergic, the odds are six to one that you are allergic.

(d) How would your doctor calculate these odds of being allergic based on what's known about the allergy test?

(e) Another doctor reviews your test results and medical record and says your odds of being allergic are really much higher, namely thirty-six to one. Briefly explain how two conscientious doctors could disagree so much. Is there a way you could determine your actual odds of being allergic?

Problem 3.
An *International Journal of Pharmacological Testing* has a policy of publishing drug trial results only if the conclusion holds at the 95% confidence level. The editors and reviewers always carefully check that any results they publish came from a drug trial that genuinely deserved this level of confidence. They are also careful to check that trials whose results they publish have been conducted independently of each other.

The editors of the Journal reason that under this policy, their readership can be confident that at most 5% of the published studies will be mistaken. Later, the editors are embarrassed—and astonished—to learn that *every one* of the 20 drug trial results they published during the year was wrong. The editors thought that because the trials were conducted independently, the probability of publishing 20 wrong results was negligible, namely, $(1/20)^{20} < 10^{-25}$.

Write a brief explanation to these befuddled editors explaining what's wrong with their reasoning and how it could be that all 20 published studies were wrong.

Hint: xkcd comic: "significant" `xkcd.com/882/`

Problem 4.
Event E is *evidence in favor* of event H when $\Pr\left[H \mid E\right] > \Pr[H]$, and it is *evidence against* H when $\Pr\left[H \mid E\right] < \Pr[H]$.

(a) Give an example of events A, B, H such that A and B are independent, both are evidence for H, but $A \cup B$ is evidence against H.

Hint: Let $\mathcal{S} = [1..8]$

(b) Prove E is evidence in favor of H iff $\overline{E}$ is evidence against H.

Supplemental Problem

Problem 5.
Describe events A, B and C that:

- satisfy the "product rule," namely,

$$\Pr[A \cap B \cap C] = \Pr[A] \cdot \Pr[B] \cdot \Pr[C],$$

- and additionally, no two out of the three events are independent.

Hint: Choose A, B, C events over the uniform probability space on [1..6].