In-Class Problems Week 11, Wed.

Problem 1.
Find the remainder of $26^{1818181}$ divided by 297.

Hint: $1818181 = (180 \cdot 10101) + 1$; use Euler's theorem.

Problem 2. **(a)** Prove that 2012^{1200} has a multiplicative inverse modulo 77.

(b) What is the value of $\phi(77)$, where ϕ is Euler's function?

(c) What is the remainder of 2012^{1200} divided by 77?

Problem 3.
Prove that for any prime p and integer $k \geq 1$,

$$\phi(p^k) = p^k - p^{k-1},$$

where ϕ is Euler's function. *Hint:* Which numbers between 0 and $p^k - 1$ *are* divisible by p? How many are there?

Note: This is proved in the text. Don't look up that proof.

Problem 4.
At one time, the Guinness Book of World Records reported that the "greatest human calculator" was a guy who could compute 13th roots of 100-digit numbers that were 13th powers. What a curious choice of tasks. . . .

In this problem, we prove

$$n^{13} \equiv n \pmod{10} \tag{1}$$

for all n.

(a) Explain why (1) does not follow immediately from Euler's Theorem.

(b) Prove that

$$d^{13} \equiv d \pmod{10} \tag{2}$$

for $0 \leq d < 10$.

(c) Now prove the congruence (1).