

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Rules for Counting



Albert R Meyer, April 17, 2013

rulescount.1

6	9	13	7
12		10	5
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15	8	11	2

Counting in Gambling

What *fraction* of poker hands are "a pair of Jacks?"
(*probability* of a pair of Jacks)



Albert R Meyer, April 17, 2013

rulescount.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Counting in Games



different chess positions
after n moves?



different positions
for a Rubik's cube?



Albert R Meyer, April 17, 2013

rulescount.3

6	9	13	7
12		10	5
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Counting in Algorithms

ops to update a data
structure (# comparisons
needed to sort n items)
steps in a computation (#
multiplies to compute d^n)



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rulescount.5

6	9	13	7
12		10	5
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Counting in **Cryptography**

possible passwords

possible keys

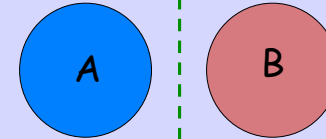


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rulescount.6

6	9	13	7
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Sum Rule



If sets A and B are **disjoint**, then

$$|A \cup B| = |A| + |B|$$



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rulescount.8

6	9	13	7
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Sum Rule

- Class has 43 women, 54 men so total enrollment = $43 + 54 = 97$
- 26 lower case letters, 26 upper case letters, and 10 digits, so # characters = $26+26+10 = 62$



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rulescount.9

6	9	13	7
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Product Rule

If there are 4 boys and 3 girls, there are

$$4 \cdot 3 = 12$$

different boy/girl couples



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rulescount.10

6	9	13	7
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Product Rule

If $|A| = m$ and $|B| = n$, then
 $|A \times B| = m \cdot n$

$$A = \{a, b, c, d\}, \quad B = \{1, 2, 3\}$$

$$A \times B = \{(a,1), (a,2), (a,3), \\ (b,1), (b,2), (b,3), \\ (c,1), (c,2), (c,3), \\ (d,1), (d,2), (d,3)\}$$



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rulescount.11

6	9	13	7
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Product Rule: Counting Strings

length-4 binary strings

$$= |B \times B \times B \times B|$$

$$= |B^4| \text{ where } B ::= \{0,1\}$$

$$= 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$$



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rulescount.12

6	9	13	7
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Product Rule: Counting Strings

length n strings
 from an alphabet of
 size m is

$$m^n$$



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rulescount.13