

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Counting in Poker

2 Pair



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

playing cards

cards have

rank: A, 2, 3, ..., 10, J, Q, K

suit: ♠, ♥, ♦, ♣

total: $13 \cdot 4 = 52$ cards



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

a 2-pair hand has

- 2 cards of some rank
- 2 cards of a second rank
- 1 card of still a third rank



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

a 2-pair hand:

$K♦, K♥, A♦, A♠, 3♣$



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

to count, choose:

- 1st pair rank (13 ranks)
- 2nd pair rank (12 ranks left)
- last card rank (11 ranks left)



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2pair.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

then choose:

- 1st pair suits $\binom{4}{2}$ sets of 2 suits
- 2nd pair suits $\binom{4}{2}$ sets of 2 suits
- last card suit (4 suits)



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2pair.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

example: choosing

K, A, 3, {♦, ♥}, {♦, ♠}, ♣

specifies 2-pair hand:



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2pair.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

example: choosing

K, A, 3, {♦, ♥}, {♦, ♠}, ♣

specifies 2-pair hand:

K♦, K♥



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2pair.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

example: choosing

$K, A, 3, \{\diamond, \heartsuit\}, \{\diamond, \spadesuit\}, \clubsuit$

specifies 2-pair hand:

$K\diamond, K\heartsuit, A\diamond, A\spadesuit$



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2pair.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

example: choosing

$K, A, 3, \{\diamond, \heartsuit\}, \{\diamond, \spadesuit\}, \clubsuit$

specifies 2-pair hand:

$K\diamond, K\heartsuit, A\diamond, A\spadesuit, 3\clubsuit$



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2pair.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands
so # 2-pair hands is

$$13 \cdot 12 \cdot 11 \cdot \binom{4}{2}! \cdot \binom{4}{2} \cdot 4$$



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2pair.11

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands
this method counts 6-tuples
[1st card ranks] × [2nd card ranks]
× [last card rank] ×
[1st card suits] × [2nd card suits]
× [last card suit]

correctly

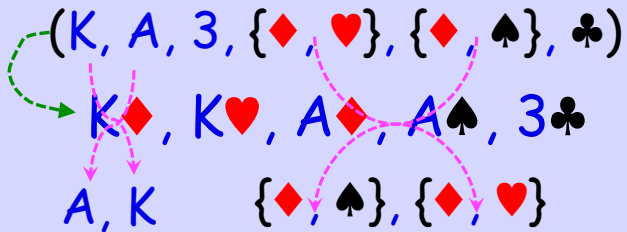


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2pair.12

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands
but the correspondence to
2-pair hands is **not a bijection**:



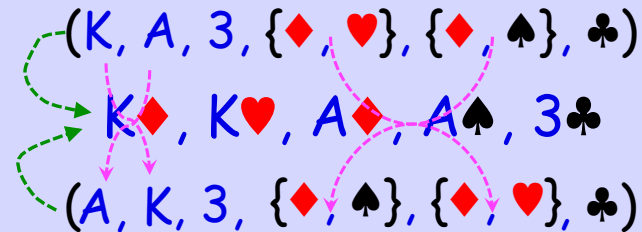
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2pair.13

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands
but the correspondence to
2-pair hands is **not a bijection**:



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2pair.14

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

to count, choose: **the bug**

- 1st pair rank (13 ranks)
- 2nd pair rank (12 ranks left)
- last card rank (11 ranks left)



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2pair.16

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

to count, choose: **the bug**

- 1st pair rank (13 ranks)
- 2nd pair rank (12 ranks left)
- last card rank (11 ranks left)

either pair might be 1st



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2pair.17

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

map from 6-tuples

$(K, A, 3, \{\diamond, \heartsuit\}, \{\diamond, \spadesuit\}, \clubsuit)$

to 2-pair hands

$K\diamond, K\heartsuit, A\diamond, A\spadesuit, 3\clubsuit$

is 2-to-1



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2pair.18

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

so # 2-pair hands is

$$13 \cdot 12 \cdot 11 \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 4$$

NO!



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2pair.19

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

counting 2-pair poker hands

so # 2-pair hands is really

$$\frac{1}{2} \cdot 13 \cdot 12 \cdot 11 \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 4$$



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2pair.20