Mathematics for Computer Science

MIT 6.042J/18.062J

Counting in Poker 2 Pair

playing cards

cards have

rank: A, 2, 3,…,10, J, Q, K

suit: ♠, ♥, ♦, ♣

total: 13·4 = 52 cards

counting 2-pair poker hands

a 2-pair hand has

- 2 cards of some rank
- 2 cards of a second rank
- 1 card of still a third rank

counting 2-pair poker hands

a 2-pair hand:

K♦, K♥, A♦, A♠, 3♣

counting 2-pair poker hands

to count, choose:

- 1st pair rank (13 ranks)
- 2nd pair rank (12 ranks left)
- last card rank (11 ranks left)

counting 2-pair poker hands

then choose:

- 1st pair suits $\binom{4}{2}$ sets of 2 suits
- 2nd pair suits $\binom{4}{2}$ sets of 2 suits
- last card suit (4 suits)

counting 2-pair poker hands

example: choosing

K, A, 3, {♦, ♥}, {♦, ♠}, ♣

specifies 2-pair hand:

counting 2-pair poker hands

example: choosing

K, A, 3, {♦, ♥}, {♦, ♠}, ♣

specifies 2-pair hand:

K♦, K♥

counting 2-pair poker hands

example: choosing

K, A, 3, {♦, ♥}, {♦, ♠}, ♣

specifies 2-pair hand:

K♦, K♥, A♦, A♠

counting 2-pair poker hands

example: choosing

K, A, 3, {♦, ♥}, {♦, ♠}, ♣

specifies 2-pair hand:

K♦, K♥, A♦, A♠, 3♣

counting 2-pair poker hands

so # 2-pair hands is

$$13 \cdot 12 \cdot 11 \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 4$$

NO!

counting 2-pair poker hands

this method counts 6-tuples

[1st card ranks]×[2nd card ranks]

× [last card rank] ×

[1st card suits] × [2nd card suits]

× [last card suit]

correctly

counting 2-pair poker hands

but the correspondence to 2-pair hands is not a bijection:

(K, A, 3, {♦, ♥}, {♦, ♠}, ♣)

K♦, K♥, A♦, A♠, 3♣

A, K {♦, ♠}, {♦, ♥}

counting 2-pair poker hands

but the correspondence to 2-pair hands is not a bijection:

(K, A, 3, {♦, ♥}, {♦, ♠}, ♣)

K♦, K♥, A♦, A♠, 3♣

(A, K, 3, {♦, ♠}, {♦, ♥}, ♣)

counting 2-pair poker hands

to count, choose: the bug

- 1st pair rank (13 ranks)
- 2nd pair rank (12 ranks left)
- last card rank (11 ranks left)

counting 2-pair poker hands

to count, choose: the bug

- 1st pair rank (13 ranks)
- 2nd pair rank (12 ranks left)
- last card rank (11 ranks left)

either pair might be 1st

counting 2-pair poker hands

map from 6-tuples

(K, A, 3, {♦, ♥}, {♦, ♠}, ♣)

to 2-pair hands

K♦, K♥, A♦, A♠, 3♣

is 2-to-1

counting 2-pair poker hands

so # 2-pair hands is

$$13 \cdot 12 \cdot 11 \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 4$$

NO!

counting 2-pair poker hands

so # 2-pair hands is really

$$\frac{1}{2} \cdot 13 \cdot 12 \cdot 11 \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 4$$