Counting in Poker

2 Pair

Counting 2-pair poker hands

a 2-pair hand has
• 2 cards of some rank
• 2 cards of a second rank
• 1 card of still a third rank

playing cards

cards have

rank: A, 2, 3,...,10, J, Q, K

suit: ♠, ♥, ♦, ♣

total: 13 \cdot 4 = 52 cards

Counting 2-pair poker hands

a 2-pair hand:

K♦, K♥, A♦, A♠, 3♣
counting 2-pair poker hands

to count, choose:
• 1st pair rank (13 ranks)
• 2nd pair rank (12 ranks left)
• last card rank (11 ranks left)

then choose:
• 1st pair suits \( \binom{4}{2} \) sets of 2 suits
• 2nd pair suits \( \binom{4}{2} \) sets of 2 suits
• last card suit (4 suits)

example: choosing

\( \text{K, A, 3, \{\spadesuit, \heartsuit\}, \{\spadesuit, \diamondsuit\}, \clubsuit} \)

specifies 2-pair hand:

\( \text{K\spadesuit, K\heartsuit} \)
counting 2-pair poker hands

eexample: choosing

\[K, A, 3, \{\diamondsuit, \heartsuit\}, \{\diamsuit, \spadesuit\}, \clubsuit\]
specifies 2-pair hand:

\[K\diamondsuit, K\heartsuit, A\diamondsuit, A\spadesuit\]

so \# 2-pair hands is

\[\frac{13 \cdot 12 \cdot 11 \cdot 4 \cdot 4}{2\cdot2}\]

\[\text{NO!}\]

\[\text{this method counts 6-tuples} \times [1\text{st card ranks}] \times [2\text{nd card ranks}] \times [\text{last card rank}] \times [1\text{st card suits}] \times [2\text{nd card suits}] \times [\text{last card suit}]\]

\[\text{correctly}\]
counting 2-pair poker hands
but the correspondence to
2-pair hands is not a bijection:

\[(K, A, 3, \{\diamondsuit, \heartsuit\}, \{\spadesuit, \clubsuit\}, \spadesuit)\]

\[K \diamondsuit, K \heartsuit, A \diamondsuit, A \spadesuit, 3 \spadesuit\]

\[A, K \{\diamondsuit, \spadesuit\}, \{\spadesuit, \heartsuit\}\]

to count, choose:

- 1st pair rank (13 ranks)
- 2nd pair rank (12 ranks left)
- last card rank (11 ranks left)

either pair might be 1st
counting 2-pair poker hands
map from 6-tuples
(K, A, 3, {♦, ♥}, {♦, ♠}, ♣)
to 2-pair hands
K♦, K♥, A♦, A♠, 3♣
is 2-to-1

counting 2-pair poker hands
so # 2-pair hands is
13 \cdot 2 \cdot 1 \cdot \frac{4}{2} \cdot \frac{4}{2} \cdot 4

NO!

counting 2-pair poker hands
so # 2-pair hands is really
\frac{1}{2} \cdot 13 \cdot 12 \cdot 11 \cdot \frac{4}{2} \cdot \frac{4}{2} \cdot 4