Mathematics for Computer Science
MIT 6.042J/18.062J

Proof by Contradiction

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Is $\sqrt[3]{1332} \leq 11$?

If so, $1332 \leq 1331$

That's not true, so

$$\sqrt[3]{1332} > 11$$

Proof by Contradiction

If an assertion implies something false, then the assertion itself must be false!

Proof by Contradiction

Theorem: $\sqrt{2}$ is irrational.

- Suppose $\sqrt{2}$ was rational
- So have n, d integers without common prime factors such that
$$\sqrt{2} = \frac{n}{d}$$
- We will show that n & d are both even. This contradicts no common factor.

Proof by Contradiction

Theorem: $\sqrt{2}$ is irrational.

$$\sqrt{2} = \frac{n}{d}$$

$$\sqrt{2}d = n$$

$$2d^2 = n^2$$

So n is even

QED

so can assume

$$n = 2k$$

$$n^2 = 4k^2$$

$$2d^2 = 4k^2$$

$$d^2 = 2k^2$$

So d is even

Quickie

Proof assumes that

if n^2 is even, then n is even.

Why is this true?