

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
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Proof by Contradiction



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contradiction.1

6	9	13	7
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Proof by Contradiction

Is $\sqrt[3]{1332} \leq 11$?

If so, $1332 \leq 1331$

That's **not true**, so

$\sqrt[3]{1332} > 11$



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contradiction.2

6	9	13	7
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Proof by Contradiction

If an assertion implies something **false**, then the assertion itself must be **false**!



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contradiction.3

6	9	13	7
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Proof by Contradiction

Theorem: $\sqrt{2}$ is irrational.

- Suppose $\sqrt{2}$ was **rational**
- So have n, d integers **without common prime factors** such that

$$\sqrt{2} = \frac{n}{d}$$
- We will show that n & d are **both even**.
This **contradicts no common factor**.



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contradiction.4

6	9	13	7
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Proof by Contradiction

Theorem: $\sqrt{2}$ is irrational.

so can assume

$$\begin{aligned}\sqrt{2} &= \frac{n}{d} \\ \sqrt{2}d &= n \\ 2d^2 &= n^2 \\ \text{So } n &\text{ is even}\end{aligned}$$

QED

$$\begin{aligned}n &= 2k \\ n^2 &= 4k^2 \\ 2d^2 &= 4k^2 \\ d^2 &= 2k^2 \\ \text{So } d &\text{ is even}\end{aligned}$$



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contradiction.5

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Quickie

Proof assumes that
if n^2 is even, then n is even.
Why is this true?



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contradiction.6