Slide 1

Mathematics for Computer Science
MIT 6.042J/18.062J

Monty Hall
Conditional
Probability

Albert R Meyer,   May 3, 2013

condmonty.1

Slide 2

Mathematics for Computer Science
MIT 6.042J/18.062J

Monty Hall
Conditional
Probability
often confusing
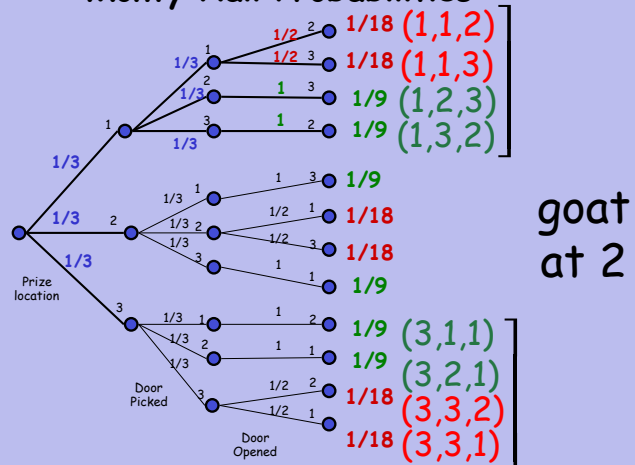
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condmonty.2

Slide 3

Monty Hall Probabilities

goat
at 2

Prize location

Door Picked

Door Opened

1/18 (1,1,2)
1/18 (1,1,3)
1/9 (1,2,3)
1/9 (1,3,2)
1/9
1/18
1/18
1/9
1/9 (3,1,1)
1/9 (3,2,1)
1/18 (3,3,2)
1/18 (3,3,1)

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condmonty.3

Slide 4

Monty Hall Probabilities

prize at 1

goat
at 2

Prize location

Door Picked

Door Opened

1/18 (1,1,2)
1/18 (1,1,3)
1/9 (1,2,3)
1/9 (1,3,2)
1/9
1/18
1/18
1/9
1/9 (3,1,1)
1/9 (3,2,1)
1/18 (3,3,2)
1/18 (3,3,1)

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condmonty.4

1
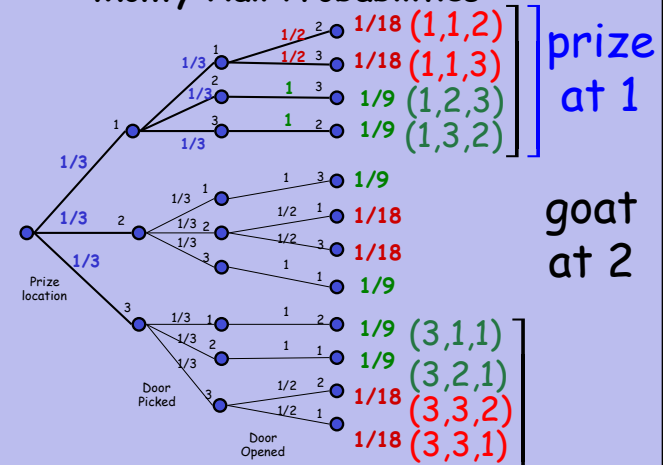
Conditional Probability: Monty Hall

$$Pr[\ \text{prize at 1}\ |\ \text{goat at 2}]$$

$$= \frac{1}{2} \quad \text{Really!}$$

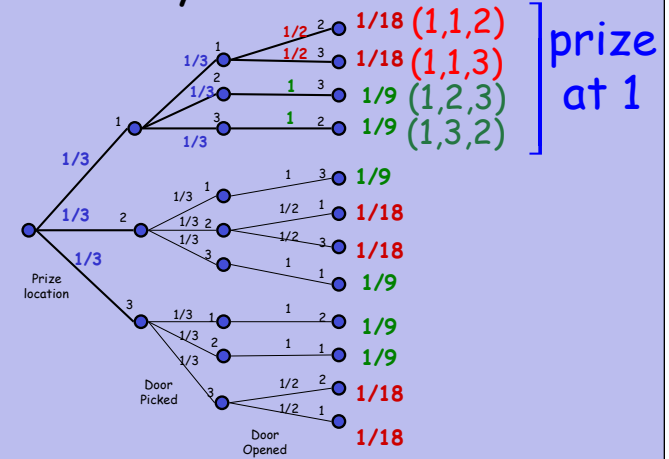Albert R Meyer,   May 3, 2013   condmonty.5

Monty Hall Probabilities

1/18 (1,1,2)
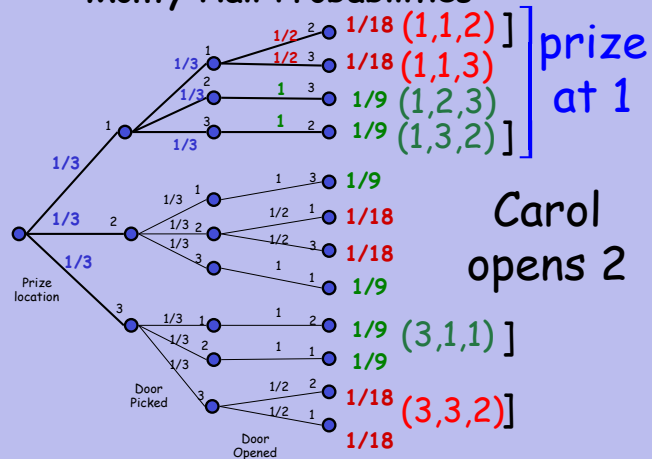1/18 (1,1,3) } prize at 1
1/9 (1,2,3)
1/9 (1,3,2)

1/9
1/18
1/18
1/9
1/9
1/9
1/18
1/18

Prize location
Door Picked
Door Opened

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Monty Hall Probabilities

1/18 (1,1,2)]
1/18 (1,1,3) } prize at 1
1/9 (1,2,3)
1/9 (1,3,2)]

1/9
1/18
1/18
1/9

Carol opens 2

1/9 (3,1,1)]
1/9
1/18 (3,3,2)]
1/18

Prize location
Door Picked
Door Opened

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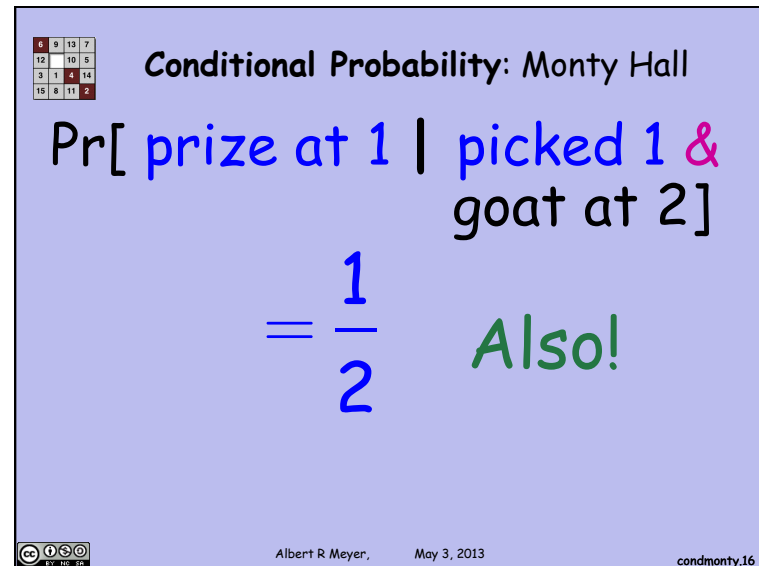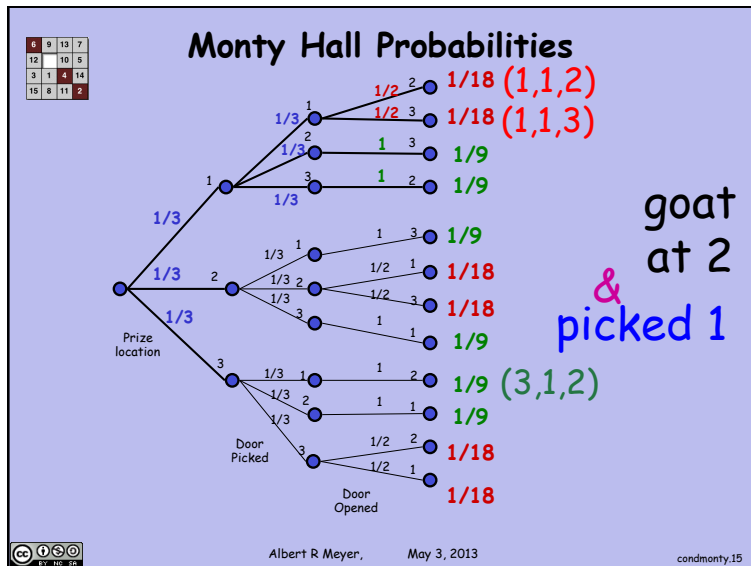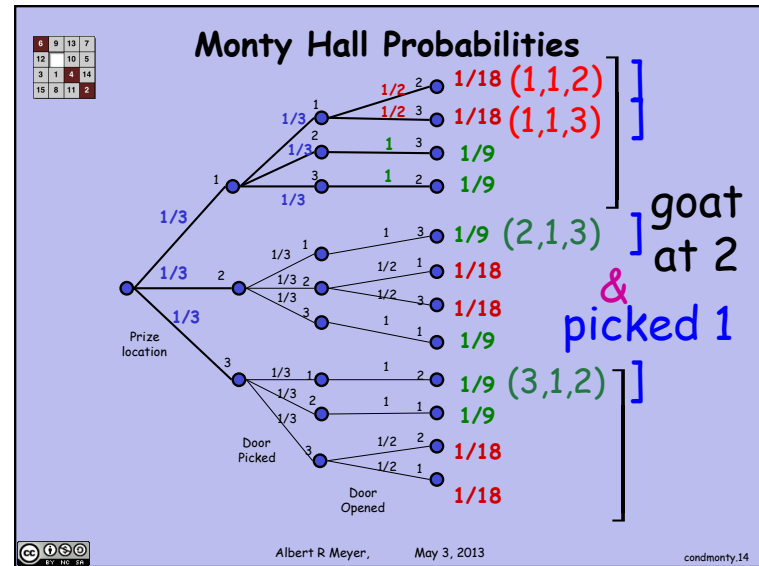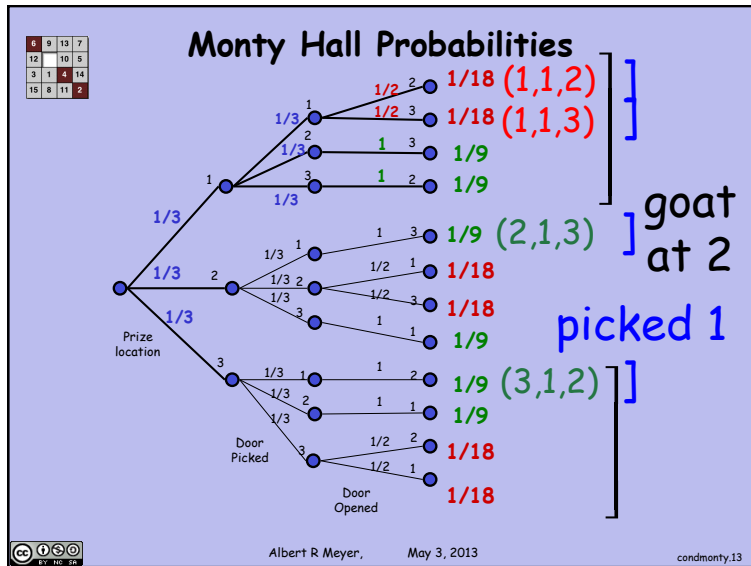
Conditional Probability: Monty Hall

$$Pr[\ \text{prize at 1}\ |\ \text{Carol opens 2}]$$

$$= \frac{1}{2} \quad \text{Likewise!}$$

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2

Monty Hall Probabilities

1/18 (1,1,2)
1/18 (1,1,3)
1/9
1/9
goat at 2
1/9 (2,1,3)
1/18
1/18
1/9
picked 1
1/9 (3,1,2)
1/9
1/18
1/18

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Monty Hall Probabilities

1/18 (1,1,2)
1/18 (1,1,3)
1/9
1/9
goat at 2
&
picked 1
1/9 (2,1,3)
1/18
1/18
1/9
1/9 (3,1,2)
1/9
1/18
1/18

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Monty Hall Probabilities

1/18 (1,1,2)
1/18 (1,1,3)
1/9
1/9
goat at 2
&
picked 1
1/9
1/18
1/18
1/9
1/9 (3,1,2)
1/9
1/18
1/18

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Conditional Probability: Monty Hall

Pr[prize at 1 | picked 1 & goat at 2]

$$= \frac{1}{2} \quad \text{Also!}$$

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3

Stick or Switch?

Seems the contestant may as well **stick**, since the probability is 1/2 given what he knows when he chooses. Wait! contestant knows more than what door he picked & where a goat is, he knows

what door Carol opened!

Conditional Probability: Monty Hall

So until now, we have been conditioning on the wrong events —a common blunder.
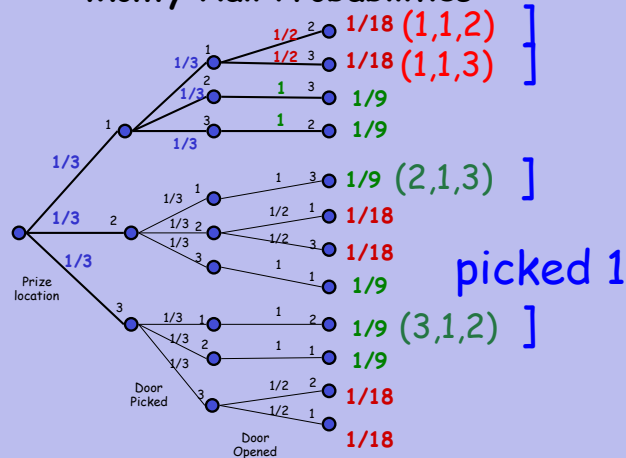
Using the correct one:

Monty Hall Probabilities

picked 1

Monty Hall Probabilities

opened 2

picked 1

4

Slide 1 (condmonty.25)

Monty Hall Probabilities

1/2 → 2 1/18 (1,1,2)]]
1/2 → 3 1/18 (1,1,3)]
1 → 3 1/9
1 → 1/9] opened 2

1/9 (2,1,3)]
1/2 → 1 1/18
1/2 → 3 1/18 &
1 → 1/9 picked 1

1/9 (3,1,2)]]
1 → 1/9
1/2 → 2 1/18
1/2 → 1 1/18]

1/3, 1/3, 1/3 (Prize location)

Prize location

Door Picked

Door Opened

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Slide 2 (condmonty.26)

Monty Hall Probabilities

1/2 → 2 1/18 (1,1,2)]]
1/2 → 3 1/18
1 → 3 1/9
1 → 1/9 opened 2

1/9
1/2 1/18
1/2 1/18 &
1 1/9 picked 1

1/9 (3,1,2)]]
1 1/9
1/2 → 2 1/18
1/2 → 1 1/18

Prize location

Door Picked

Door Opened

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Slide 3 (condmonty.27)

Monty Hall Probabilities

1/2 → 2 1/18 (1,1,2)
1/2 → 3 1/18
1 → 3 1/9
1 → 1/9 opened 2

1/9
1/2 1/18
1/2 1/18 &
1 1/9 picked 1

1/9 (3,1,2)
1 1/9
1/2 → 2 1/18
1/2 → 1 1/18

Prize location

Door Picked

Door Opened

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Slide 4 (condmonty.29)
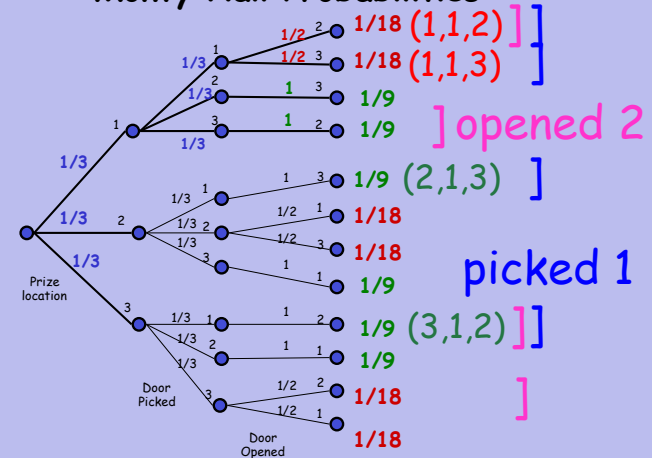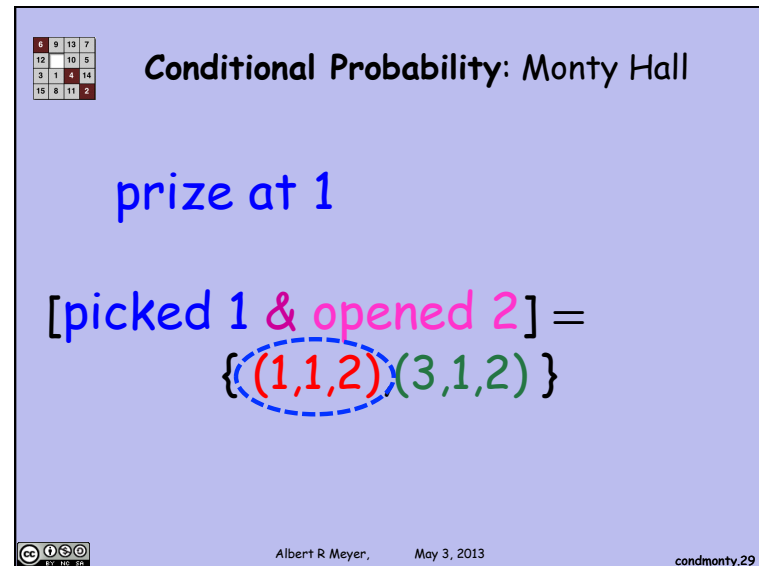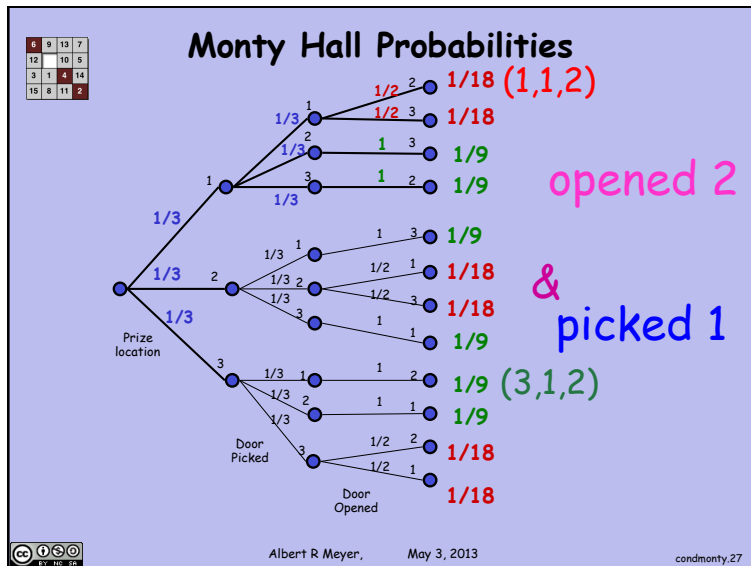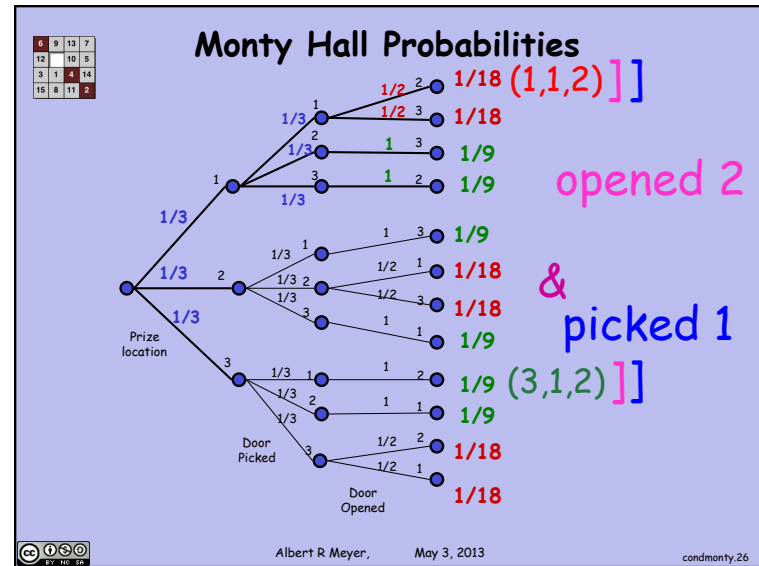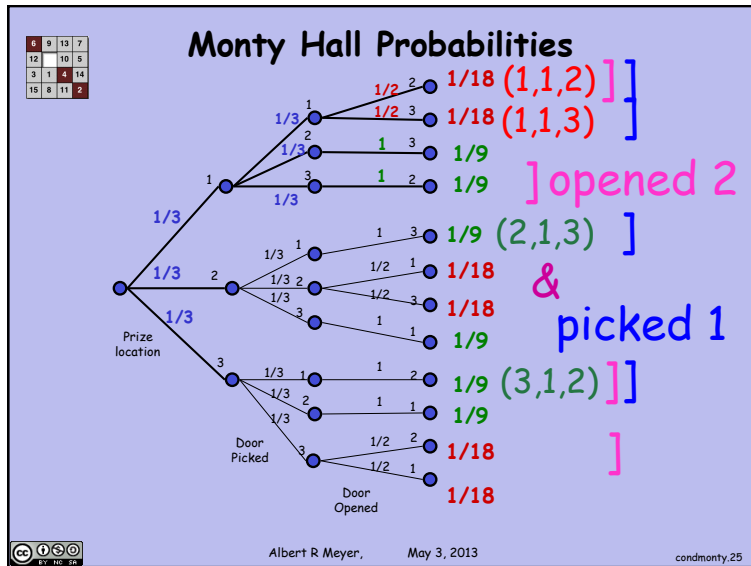
Conditional Probability: Monty Hall

prize at 1

[picked 1 & opened 2] =
{ (1,1,2) (3,1,2) }

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Conditional Probability: Monty Hall

Pr[prize at 1 | picked 1 &
opened 2]

[picked 1 & opened 2] =
{ (1,1,2) (3,1,2) }

Pr=1/18   Pr=1/9

Conditional Probability: Monty Hall

Pr[prize at 1 | picked 1 &
opened 2]

$$= \frac{1/18}{1/18 + 1/9}$$

Conditional Probability: Monty Hall

Pr[prize at 1 | picked 1 &
opened 2]

$$= \frac{1/18}{1/18 + 1/9} = \frac{1}{3}$$

$$= \text{Pr[sticking wins]}$$

Stick or Switch?

Pr[prize at 1 | picked 1 &
opened 2]

$$= \frac{1/18}{1/18 + 1/9} = \frac{1}{3}$$

$$= \text{Pr[sticking wins]}$$

6

Switch!

By conditioning on everything the contestant knows, we've finally confirmed what we learned earlier:

$$\Pr[\text{switching wins}] = \frac{2}{3}$$

The 4 Step Method

It's easy to how so many smart people get confused by Monty Hall.

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It's easy to how so many smart people get confused by Monty Hall. Finding the right event to condition on can be tricky.

The 4 Step Method

It's easy to how so many smart people get confused by Monty Hall. Finding the right event to condition on can be tricky. The 4 step method is a good fall back approach.