Mathematics for Computer Science

MIT 6.042J/18.062J

Matching Birthdays

Birthday Pairs

P ::= # pairs with matching b'days among n people in a d-day year

$$P = \sum_{1 \le i < j \le n} M_{ij}$$

M_{ij} ::= indicator that i^{th} & j^{th} birthdays match

Birthday Pairs

$$E[M_{ij}] = 1/d$$

so by linearity of E[]

$$E[P] = \sum_{1 \le i < j \le n} E[M_{ij}] = \binom{n}{2} \cdot \frac{1}{d}$$

Birthday Pairs

Have data on 179*students

$$E[P] = \binom{179}{2} \cdot \frac{1}{365} \approx 43.6$$

*excluding 2 sets of twins

Birthday Pairs

How likely is P near 43.6?

$$\Pr[|P - 43.6| > k]$$

hard to calculate!

Variance easy to calculate!

Pairwise Independence

[Albert and Drew have same b'day]

is independent of

[David and Mike have same b'day]

that is, $M_{\text{Albert,Drew}}$ & $M_{\text{David,Mike}}$ are independent

Obvious since the b'days of Albert, Drew, David & Mike are mutually independent

Pairwise Independence

[Albert and Drew have same b'day]

is independent of

[Albert and Mike have same b'day]

that is, $M_{\text{Albert,Drew}}$ & $M_{\text{Albert,Mike}}$ are independent

(pairwise, but not 3-way:

$M_{\text{Drew,Mike}}$ depends on other two)

Birthday Pairs

$$\text{Var}[M_{ij}] = (1/365)(1 - 1/365)$$

so by prwise additivity of Var[]

$$\text{Var}[P] = \sum \text{Var}[M_{ij}] = \binom{179}{2} \text{Var}[M_{ij}]$$

$$= \binom{179}{2} \cdot \frac{1}{365} \cdot \left(1 - \frac{1}{365}\right) \approx 43.5$$

$$\sigma_P < 6.6$$

Birthday Predictions

Chebyshev:

$\Pr[43.6 \pm 2\sigma \text{ pairs}] > 1 - (1/2)^2$
$= 3/4$

32 to 55 pairs 75% of the time

We actually found 47 pairs

(29 pairs & 6 triples)

Fall '13 Matching Birthdays

179 Fall '13 students: 29 Pairs & 6 Triples

2/9	4/20	7/29	9/25
2/23	5/11	8/7	10/2
3/7	6/8	8/9	10/23
3/24	6/9	8/22	10/25
3/26	6/15	8/23	10/30
3/28	7/7	8/29	11/1
3/29	7/14	9/5	11/8
4/13	7/15	9/17	12/4
4/19	7/27	9/21	