

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
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Binomial Theorem



Albert R Meyer, April 18, 2012

lec 10W.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

expression for c_k ?

$$(1+X)^n =$$

$$c_0 + c_1 X + c_2 X^2 + \dots + c_n X^n$$

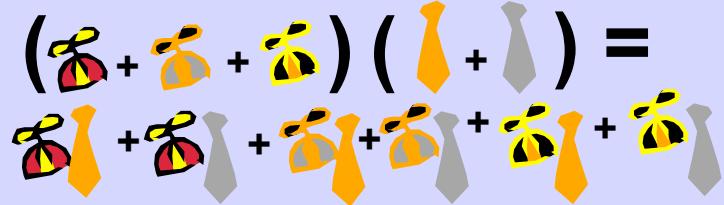


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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Polynomials Express Choices & Outcomes



Products of Sums = Sums of Products



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lec 10W.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

binomial expressions

$$(1+X)^0 = 1$$

$$(1+X)^1 = 1 + 1X$$

$$(1+X)^2 = 1 + 2X + 1X^2$$

$$(1+X)^3 = 1 + 3X + 3X^2 + 1X^3$$

$$(1+X)^4 = 1 + 4X + 6X^2 + 4X^3 + 1X^4$$



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lec 10W.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

expression for c_k ?

$$\frac{(1+X)^n}{\text{n times}} = (1+X)(1+X)(1+X)(1+X)\dots(1+X)$$

multiplying gives 2^n product terms:
 $1\dots 1 + X11X\dots X1 + 1XX\dots 1X1 + \dots + XX\dots X$
a term corresponds to selecting 1 or X from each of the n factors



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

expression for c_k ?

$$\frac{(1+X)^n}{\text{n times}} = (1+X)(1+X)(1+X)(1+X)\dots(1+X)$$

the X^k coeff, c_k , is # terms with exactly k X's selected

$$c_k = \binom{n}{k}$$



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lec 10W.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

The Binomial Formula

$$(1+X)^n = \binom{n}{0} + \binom{n}{1}X + \binom{n}{2}X^2 + \dots + \binom{n}{k}X^k + \dots + \binom{n}{n}X^n$$

binomial expression

binomial coefficients



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lec 10W.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

The Binomial Formula

$$(X+Y)^n = \binom{n}{0}Y^n + \binom{n}{1}XY^{n-1} + \binom{n}{2}X^2Y^{n-2} + \dots + \binom{n}{k}X^kY^{n-k} + \dots + \binom{n}{n}X^n$$



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lec 10W.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

The Binomial Formula

$$(X + Y)^n = \sum_{k=0}^n \binom{n}{k} X^k Y^{n-k}$$



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