

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
6.042J/18.062J

3-coloring Planar Graphs



Albert R Meyer

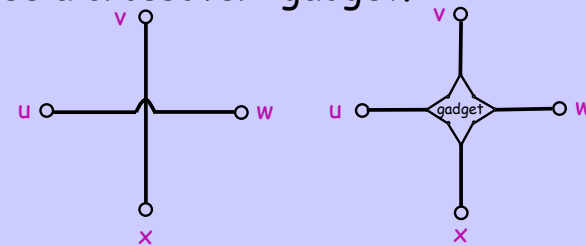
April 6, 2018

paterson.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar 3-colorings

Convert a simple graph to a planar graph with same 3-colorability.
Use a crossover "gadget."



Albert R Meyer

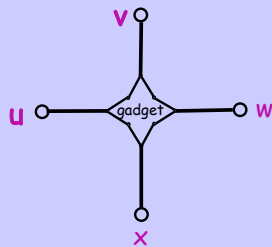
April 6, 2018

paterson.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Transmit coloring across gadget

Any assignment of colors to **u** and **v** can be extended to a 3-coloring of the gadget.



Albert R Meyer

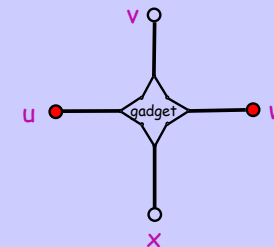
April 6, 2018

paterson.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Transmit coloring across gadget

In any 3-coloring, the colors of **u** and **w** are the same



Albert R Meyer

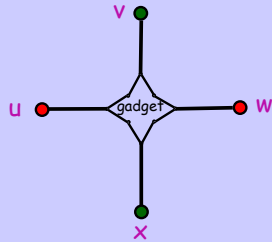
April 6, 2018

paterson.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Transmit coloring across gadget

In any 3-coloring, the colors of **u** and **w** are the same, and the colors of **v** and **x** are the same



Albert R Meyer

April 6, 2018

paterson.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Transmit coloring across gadget

...to get a **planar** graph that is 3-colorable iff the original graph is 3-colorable.



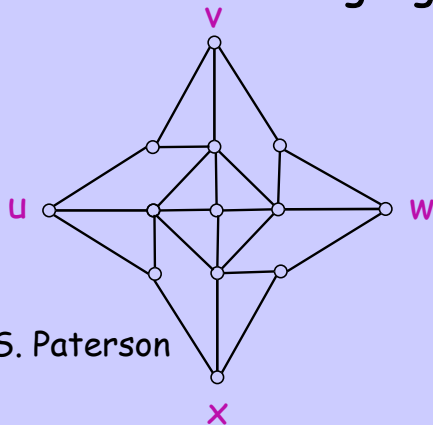
Albert R Meyer

April 6, 2018

paterson.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

3-color crossover gadget



Michael S. Paterson



Albert R Meyer

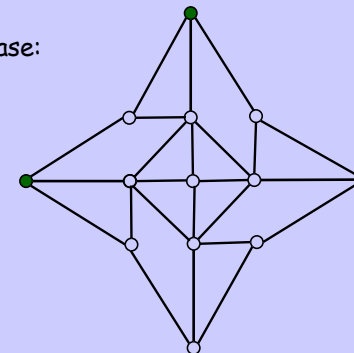
April 6, 2018

paterson.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

3-color crossover gadget

Same color case:



Albert R Meyer

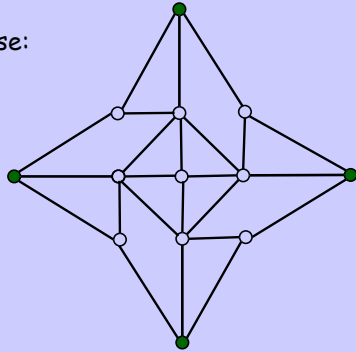
April 6, 2018

paterson.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

3-color crossover gadget

Same color case:



There is a crossover coloring



Albert R Meyer

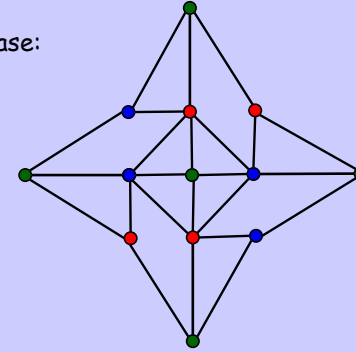
April 6, 2018

paterson.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

3-color crossover gadget

Same color case:



There is a crossover coloring



Albert R Meyer

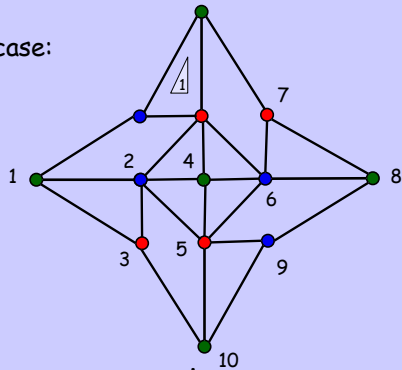
April 6, 2018

paterson.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

3-color crossover gadget

Same color case:



crossover coloring is unique



Albert R Meyer

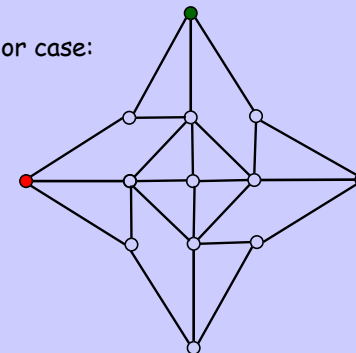
April 6, 2018

paterson.11

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

3-color crossover gadget

Different color case:



Albert R Meyer

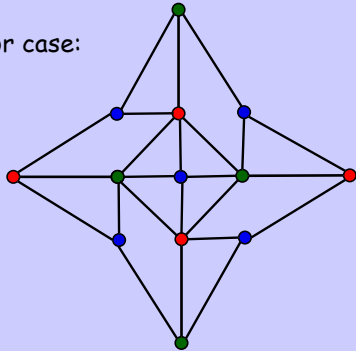
April 6, 2018

paterson.12

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

3-color crossover gadget

Different color case:



There is a crossover coloring



Albert R Meyer

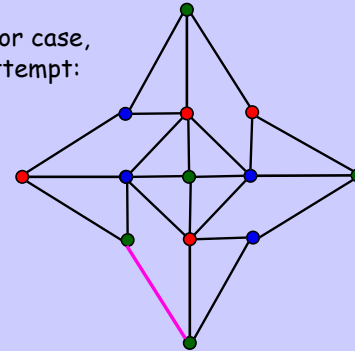
April 6, 2018

paterson.13

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

3-color crossover gadget

Different color case,
alternative attempt:



crossover coloring is unique



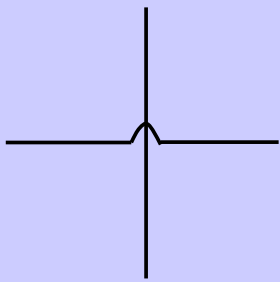
Albert R Meyer

April 6, 2018

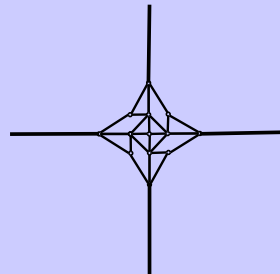
paterson.14

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Convert to planar graph



draw graph in plane
with edge crossings



replace each crossing
with gadget



Albert R Meyer

April 6, 2018

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Convert to planar graph

... yields a **planar** graph that
is 3-colorable iff original
graph is 3-colorable.

draw graph in plane
with edge crossings

replace crossing
with gadget



Albert R Meyer

April 6, 2018