Mathematics for Computer Science

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The Well Ordering Principle, III

Geometric sums

$$1 + r + r^2 + r^3 + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1}$$

Proof by WOP. Let m be smallest n with $\neq$. But $=$ for $n = 0$, so $m > 0$, and

$$1 + r + r^2 + r^3 + \cdots + r^{m-1} = \frac{r^{(m-1)+1} - 1}{r - 1}$$

Geometric sums

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Geometric sums

$$1 + r + r^2 + r^3 + \cdots + r^{m-1} = \frac{r^{m} - 1}{r - 1}$$

add r^m to both sides

$$\text{LHS} = 1 + r + r^2 + r^3 + \cdots + r^{m-1} + r^m$$

$$\text{RHS} = \frac{r^m - 1}{r - 1} + \frac{r^{m+1} - r^m}{r - 1} = \frac{r^{m+1} - 1}{r - 1}$$

so $=$ at m, contradicting $\neq$: there is no counterexample.

Well Ordering Principle Proofs

To prove $\forall n \in \mathbb{N}.\, P(n)$ using WOP:

- define set of counterexamples

$$C ::= \{n \in \mathbb{N} \mid \text{NOT } P(n)\}$$

- assume C is not empty. By WOP, have minimum element $m \in C$
- Reach a contradiction *somehow* …

usually by finding $c \in C$ with $c < m$

…or by proving $P(m)$