

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
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The Well Ordering Principle, III

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Albert R Meyer February 13, 2012 Lec 2M.1

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Geometric sums

$$1 + r + r^2 + r^3 + \dots + r^n = \frac{r^{n+1} - 1}{r - 1}$$

Proof by WOP. Let m be smallest n with \neq . But $=$ for $n = 0$, so $m > 0$, and

$$1 + r + r^2 + r^3 + \dots + r^{m-1} = \frac{r^{(m-1)+1} - 1}{r - 1}$$

Albert R Meyer February 13, 2012 Lec 2M.2

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Geometric sums

$$1 + r + r^2 + r^3 + \dots + r^n = \frac{r^{n+1} - 1}{r - 1}$$

Proof by WOP. Let m be smallest n with \neq . But $=$ for $n = 0$, so $m > 0$, and

$$1 + r + r^2 + r^3 + \dots + r^{m-1} = \frac{r^m - 1}{r - 1}$$

Albert R Meyer February 13, 2012 Lec 2M.3

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Geometric sums

$$1 + r + r^2 + r^3 + \dots + r^{m-1} = \frac{r^m - 1}{r - 1}$$

add r^m to both sides

$$\text{LHS} = 1 + r + r^2 + r^3 + \dots + r^{m-1} + r^m$$

$$\text{RHS} = \frac{r^m - 1}{r - 1} + \frac{r^{m+1} - r^m}{r - 1} = \frac{r^{m+1} - 1}{r - 1}$$

so $=$ at m , contradicting \neq : there is no counterexample.

Albert R Meyer February 13, 2012 Lec 2M.4

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Well Ordering Principle Proofs

To prove $\forall n \in \mathbb{N}. P(n)$ using WOP:

- define set of counterexamples
 $C ::= \{n \in \mathbb{N} \mid \text{NOT } P(n)\}$
- assume C is not empty. By WOP, have minimum element $m \in C$
- Reach a contradiction somehow ...
usually by finding $P(m)$ with $c < m$

Albert R Meyer February 13, 2012 Lec 2M.5