

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
6.042J/18.062J

Truth Tables Equivalence Validity



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truth-tables.1

6	9	13	7
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Truth Assignments

A **truth assignment** assigns a value **T** or **F** to each propositional variable. Computer scientists call assignment of values to variables an **environment**. If we **know the environment**, we can **find the value** of a propositional formula.



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6	9	13	7
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Evaluation in an Environment

Example: Suppose environment, v , assigns
 $v(P) = T$, $v(Q) = T$, $v(R) = F$.

Truth value of

$(\text{NOT}(P \text{ AND } Q)) \text{ OR } (R \text{ XOR NOT}(Q))$

F T T T F F F F T



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truth-tables.3

6	9	13	7
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Equivalence

Two propositional formulas are **equivalent** iff they have the **same** truth value in **all** environments.



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truth-tables.4

6 9 13 7
12 10 5
3 1 4 14
15 8 11 2

DeMorgan's Law

$\overline{P \text{ OR } Q}$ equiv to $\overline{P} \text{ AND } \overline{Q}$

P	Q	NOT(PORQ)	\overline{P} AND \overline{Q}
T	T	F	T
T	F	F	T
F	T	F	T
F	F	T	F

truth-tables.5

6 9 13 7
12 10 5
3 1 4 14
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DeMorgan's Law

$\overline{P \text{ OR } Q}$ equiv to $\overline{P} \text{ AND } \overline{Q}$

P	Q	NOT(PORQ)	\overline{P} AND \overline{Q}
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

Same final column, so equivalent
-- proof by Truth Table

truth-tables.6

6 9 13 7
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Definition of IFF

The value of (P IFF Q) is T iff P and Q have the same truth value.

Truth Table for IFF

P	Q	P IFF Q
T	T	T
T	F	F
F	T	F
F	F	T

truth-tables.7

6 9 13 7
12 10 5
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Satisfiability & Validity

A formula is **satisfiable** iff it is **true** in **some** environment.

A formula is **valid** iff it is **true** in **all** environments.

truth-tables.8

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Satisfiability & Validity

satisfiable: $P, \text{NOT}(P)$
 not satisfiable: $(P \text{ AND } \text{NOT}(P))$
 valid: $(P \text{ OR } \text{NOT}(P))$



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truth-tables.10

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Equivalence & Validity

G and H are equivalent exactly when $(G \text{ IFF } H)$ is valid



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Verifying Valid, Satisfiable

Truth table size **doubles** with each additional variable --**exponential growth**. Makes truth tables impossible when there are hundreds of variables. (In current digital circuits, there are millions of variables.)



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truth-tables.12

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Fast Test for SAT?

The $P = NP?$ question is equivalent to asking if there is an "efficient" (polynomial rather than exponential time) procedure to check **satisfiability (SAT)**.



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truth-tables.13

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SAT versus VALID

To check that G is **valid**,
can check that
 $\text{NOT}(G)$ is **not satisfiable**.
So checking for one is
equally difficult (or easy)
as checking for the other.



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truth-tables.14