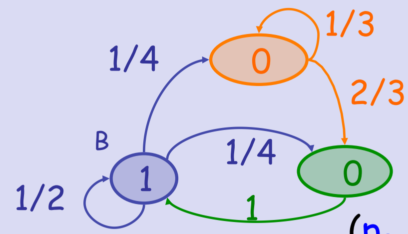
Mathematics for Computer Science

MIT 6.042J/18.062J

Stationary Distributions

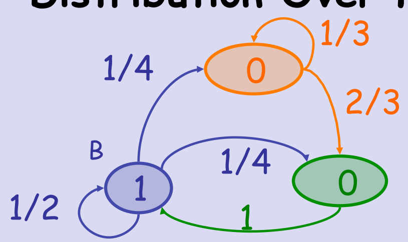
Distribution Over Nodes



Suppose you start at B: (p_B, p_O, p_G) $(1, 0, 0)$

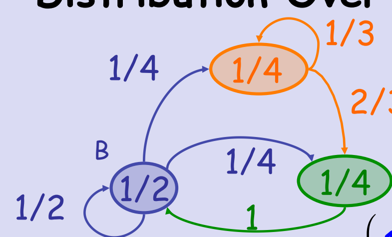
What are p'_B, p'_O, p'_G after 1 step?

Distribution Over Nodes



Dist after 1 step: (p'_B, p'_O, p'_G), only get places from B, so $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$

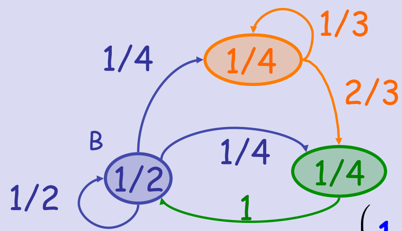
Distribution Over Nodes



Dist after 1 step: $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$

Dist after 2 steps: (p''_B, p''_O, p''_G)

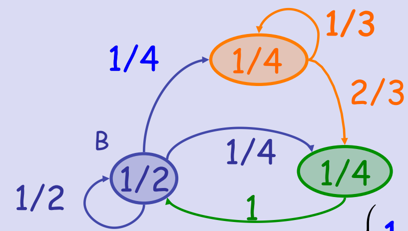
Distribution Over Nodes



Dist after 1 step: $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$

p''_O = Pr[B to O|at B]·p'_B
+ Pr[O to O|at O]·p'_O
+ Pr[G to O|at G]·p'_G

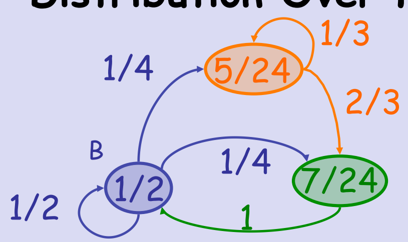
Distribution Over Nodes



Dist after 1 step: $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$

p''_O = 1/4 · 1/2
+ 1/3 · 1/4 = 5/24
+ 0 · 1/4

Distribution Over Nodes



distribution after 2 steps:
(p''_B, p''_O, p''_G)
$\left(\frac{1}{2}, \frac{5}{24}, \frac{7}{24}\right)$

Linear Algebra

The edge probability matrix for a random walk graph is the same as the adjacency matrix, using edge probabilities instead of zeroes and ones.

Linear Algebra

the edge probability matrix

$$M ::= \begin{pmatrix} \Pr[B \to B] & \Pr[B \to O] & \Pr[B \to G] \\ \Pr[O \to B] & \Pr[O \to O] & \Pr[O \to G] \\ \Pr[G \to B] & \Pr[G \to O] & \Pr[G \to G] \end{pmatrix}$$

Linear Algebra

the edge probability matrix

$$M ::= \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ \Pr[O \to B] & \Pr[O \to O] & \Pr[O \to G] \\ \Pr[G \to B] & \Pr[G \to O] & \Pr[G \to G] \end{pmatrix}$$

Linear Algebra

the edge probability matrix

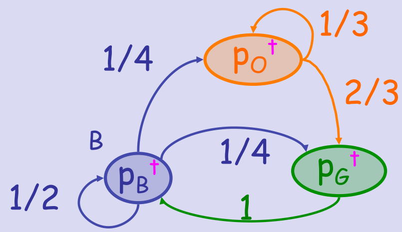
$$M ::= \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 1/3 & 2/3 \\ 1 & 0 & 0 \end{pmatrix}$$

Linear Algebra

distribution update is vector/matrix multiplication

$$(p_B, p_O, p_G) \cdot M = (p'_B, p'_O, p'_G)$$

Distribution Over Nodes



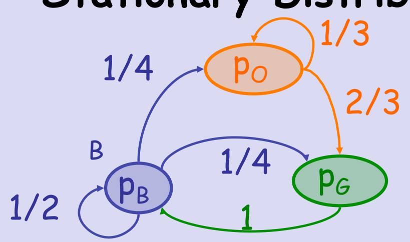
distribution after t steps?

...and as $t \to \infty$?

Linear Algebra

$$(p_B, p_O, p_G) \cdot M^t = (p_B^t, p_O^t, p_G^t)$$

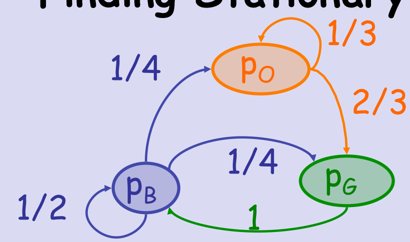
Stationary Distribution



distribution (p_B, p_O, p_G) is **stationary** if next-step distribution is the same.

What is a stationary dist. here?

Finding Stationary Dist.



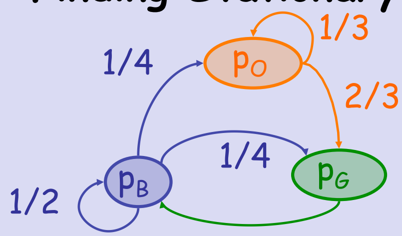
$p_B = p_B' = (1/2)p_B + 1p_G$

$p_O = p_O' = (1/4)p_B + (1/3)p_O$

$p_G = p_G' = (1/4)p_B + (2/3)p_O$

$p_B + p_O + p_G = 1$

Finding Stationary Dist.



solving for (p_B, p_O, p_G): $\left(\frac{8}{15}, \frac{3}{15}, \frac{4}{15}\right)$

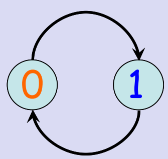
Linear Algebra

Find stationary dist vector $\vec{s}$ by solving:

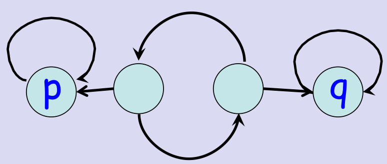
$$\vec{s} \cdot M = \vec{s}$$

$$\sum s_i = 1$$

Stationary Difficulties



does not converge to stable distribution



uncountably many stable distributions

Questions on Stationary Dist

- ∃stationary dist? — Yes (if graph finite)
- unique? — Sometimes
- converge to it from any start? — Sometimes
  - How quickly? — Varies