

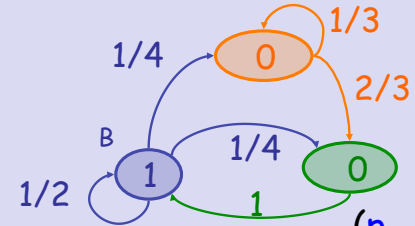
6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

# Stationary Distributions



6	9	13	7
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## Distribution Over Nodes



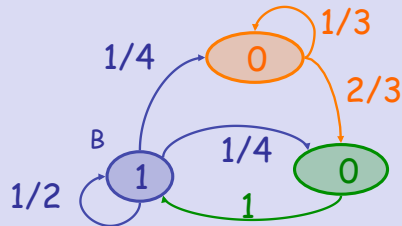
Suppose you start at B:  $(p_B, p_O, p_G)$   
 $(1, 0, 0)$

What are  $p'_B, p'_O, p'_G$  after 1 step?



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## Distribution Over Nodes

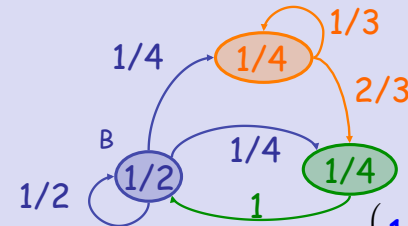


Dist after 1 step:  $(p'_B, p'_O, p'_G)$ ,  
only get places from B,  $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 4 \end{pmatrix}$   
so



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## Distribution Over Nodes

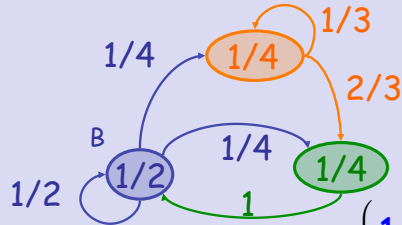


Dist after 1 step:  $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 4 \end{pmatrix}$   
Dist after 2 steps:  $(p''_B, p''_O, p''_G)$



6	9	13	7
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## Distribution Over Nodes



Dist after 1 step:

$$p''_O = \Pr[B \text{ to } O | \text{at } B] \cdot p'_B + \Pr[O \text{ to } O | \text{at } O] \cdot p'_O + \Pr[G \text{ to } O | \text{at } G] \cdot p'_G$$

$$\left( \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \right)$$



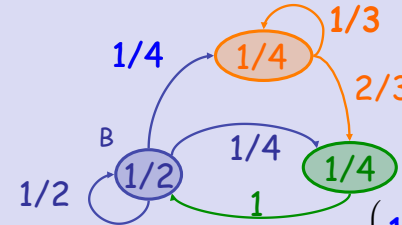
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stationary.7

6	9	13	7
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## Distribution Over Nodes



Dist after 1 step:

$$p''_O = \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{4} + 0 \cdot \frac{1}{4} = \frac{5}{24}$$



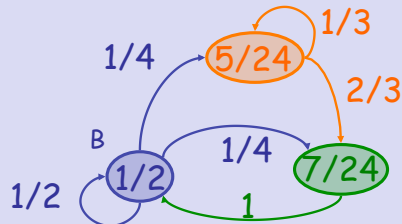
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6	9	13	7
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## Distribution Over Nodes



distribution after 2 steps:

$$(p''_B, p''_O, p''_G)$$

$$\left( \frac{1}{2}, \frac{5}{24}, \frac{7}{24} \right)$$



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## Linear Algebra

The **edge probability matrix** for a random walk graph is the same as the adjacency matrix, using edge probabilities instead of zeroes and ones.



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## Linear Algebra

the edge probability matrix

$M ::=$

$$\begin{pmatrix} \Pr[B \rightarrow B] & \Pr[B \rightarrow O] & \Pr[B \rightarrow G] \\ \Pr[O \rightarrow B] & \Pr[O \rightarrow O] & \Pr[O \rightarrow G] \\ \Pr[G \rightarrow B] & \Pr[G \rightarrow O] & \Pr[G \rightarrow G] \end{pmatrix}$$



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6	9	13	7
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## Linear Algebra

the edge probability matrix

$M ::=$

$$\begin{pmatrix} 1/2 & 1/4 & 1/4 \\ \Pr[O \rightarrow B] & \Pr[O \rightarrow O] & \Pr[O \rightarrow G] \\ \Pr[G \rightarrow B] & \Pr[G \rightarrow O] & \Pr[G \rightarrow G] \end{pmatrix}$$



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stationary.15

6	9	13	7
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## Linear Algebra

the edge probability matrix

$$M ::= \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 1/3 & 2/3 \\ 1 & 0 & 0 \end{pmatrix}$$



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## Linear Algebra

distribution update is  
vector/matrix multiplication

$$\begin{aligned} (p_B, p_O, p_G) \cdot M \\ = (p'_B, p'_O, p'_G) \end{aligned}$$

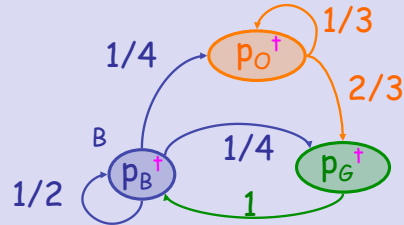


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## Distribution Over Nodes



distribution after  $t$  steps?  
...and as  $t \rightarrow \infty$ ?



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stationary.19

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## Linear Algebra

$$(p_B, p_O, p_G) \cdot M^t = (p_B^t, p_O^t, p_G^t)$$

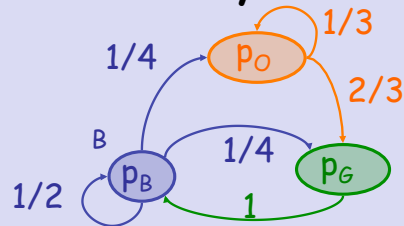


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stationary.20

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## Stationary Distribution



distribution  $(p_B, p_O, p_G)$  is **stationary** if next-step distribution is the same.  
What is a stationary dist. here?

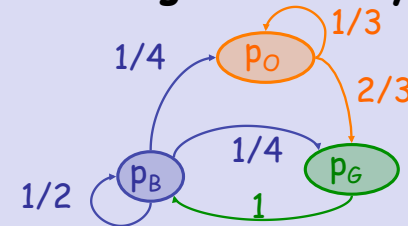


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stationary.21

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## Finding Stationary Dist.



$$\begin{aligned} p_B &= p_B' = (1/2)p_B + 1p_G \\ p_O &= p_O' = (1/4)p_B + (1/3)p_O \\ p_G &= p_G' = (1/4)p_B + (2/3)p_O \\ p_B + p_O + p_G &= 1 \end{aligned}$$

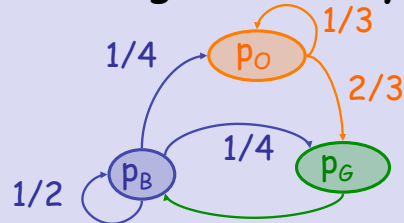


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stationary.22

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## Finding Stationary Dist.



solving for  $(p_B, p_O, p_G)$ :  $\left( \frac{8}{15}, \frac{3}{15}, \frac{4}{15} \right)$



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stationary.23

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## Linear Algebra

Find stationary dist vector  $\vec{s}$  by solving:

$$\vec{s} \cdot M = \vec{s}$$

$$\sum s_i = 1$$



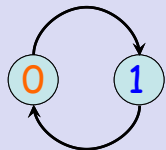
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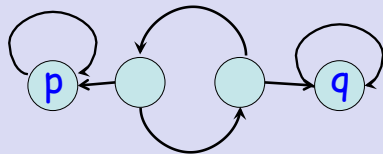
stationary.24

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## Stationary Difficulties



does not converge to stable distribution



uncountably many stable distributions



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stationary.25

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## Questions on Stationary Dist

- $\exists$  stationary dist?
- unique?
- converge to it from any start?
- How quickly?

Yes  
(if graph finite)

Sometimes

Sometimes

Varies



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stationary.26